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Stability analysis in a car-following model with reaction-time delay and delayed feedback control

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HIGHLIGHTS

- Stability analysis of a controlled car-following model with driver's reaction-time delay is studied.
- A delayed-feedback control of both headway and velocity differences is proposed to guarantee the stability.
- An upper bound on explicit time delay is determined according to the response of desired acceleration.
- The controller can suppress traffic congestion by choosing proper feedback gains and time delay.

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ABSTRACT

The delayed feedback control in terms of both headway and velocity differences has been proposed to guarantee the stability of a car-following model including the reaction-time delay of drivers. Using Laplace transformation and transfer function, the stable condition is derived and appropriate choices of time delay and feedback gains are designed to stabilize traffic flow. Meanwhile, an upper bound on explicit time delay is determined with respect to the response of desired acceleration. To ensure the string stability, the explicit time delay cannot over its upper bound. Numerical simulations indicate that the proposed control method can restraint traffic congestion and improve control performance.

1. Introduction

To achieve the objectives of smooth road traffic with effective road traffic control and the improvements of traffic efficiency and energy economy, what is needed in the context of transport includes a set of coherent transport control measures based on the identification of road traffic pattern. Microscopic traffic models (e.g. car-following models) focus on the describing of detailed behaviors of individual vehicles, where vehicles' motions are described by differential equations, provide a valuable approach to understand and explain different road traffic phenomena and characteristics. Car-following theory, which consider how one driver follows his/her immediately ahead car, has been well studied several decades with both theoretical analyses and empirical investigation, e.g., the early linear car-following model proposed by Pipes [1], the nonlinear car-following models approaches [2–6], the role in intelligent transportation system [7–11]. Moreover, some improved versions of car-following models have been developed. For example, an optimal velocity model (OVM) [12] was introduced by Bando et al. to describe many important characteristics of real traffic flows, such as the evolution of traffic jam, the density–flux relationship, and the stop-and-go traffic waves. The car-following models with the reaction-time delay

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of drivers [13–16] were presented to reveal the influence of delay on the traffic flow, the Hopf bifurcation and the chaotic behaviors. It was found that the delays destabilized traffic flow and induced the formation of traffic jam. Therefore, the reaction-time delay of drivers plays an important role in the evolution and stability of traffic flow. It is necessary to include the reaction-time delay of drivers in the modeling of traffic flow.

Traffic congestion mitigation is an important problem from the viewpoints of both economy and environment protection. The research to provide various measures to alleviate congestion, improve air quality, and increase the efficiency of the transport system has been well considered [17]. Theoretically, the formation of traffic congestion is explained as the loss of linear stability of the uniform flow equilibrium to perturbations. Hence, the desired goal of traffic flow control is to achieve equilibrium and stability. Asymptotic stability of traffic flow is concerned with the manner in which a fluctuation in the motion of any vehicle, say the lead vehicle of a platoon, is propagated through a line of vehicles [18]. To guarantee the linear stability, different control strategies to mitigate traffic congestion and improve efficiency of the traffic flow have been proposed. For example, adaptive cruise control (ACC) devices can gather the distance and velocity difference from the driver in front and the corresponding actuators can carry out the required actions (e.g. braking or acceleration). Now ACC controllers are equipped in some intelligent vehicles and could be beneficial to traffic congestion avoidance [19–21]. A decentralized delayed feedback control (DDFC) [22,23] was proposed to suppress traffic jam in a coupled map car-following model and the OVM. Based on DDFC, some improved control methods [24–29] were developed to suppress traffic congestion and stabilized the traffic flow. Especially, the delayed-velocity feedback signal was used by Davis [24] in a car-following model to guarantee the string stability. A delayed feedback control considering displacement and velocity differences was proposed by Jin and Hu [25] to suppress traffic jam in an OVM and improve the control performance. The control strategies to guarantee the exponential stability [30] were presented in a car-following model with human memory effects and automated headway compensation. However, most current studies only deal with the control strategies of car-following models without considering the drivers' reaction-time delay. In practice, the effects of time delay (e.g. driver's reaction-time delay) should be considered in designing the next generation of controllers. Thus, this paper attempts to design a delayed feedback control for a car-following model with the driver's reaction-time delay, which can suppress the destabilization effect induced by reaction-time delay of drivers.

The paper is organized as follows. Section 2 presents the stability conditions of the car-following model with driver's reaction-time delay. A delayed feedback control of both headway and velocity differences is proposed to stabilize the unstable traffic flow. An upper bound of the explicit delay time is determined by feedback gains, reaction-time delay and time delay in controller. In Section 3, numerical simulations are presented to verify the theoretical results. Conclusions are drawn in Section 4.

2. Controlled car-following model and its stability

2.1. Stability of the controlled model

A car-following model is the so-called OVM with driver's reaction-time delay introduced by Bando et al. [13], which describes the motion of a N-vehicle system on a circular lane. The equation is written as

$$\ddot{x}_n(t+\tau) = \alpha\left\{U(\Delta x_n(t)) - \dot{x}_n(t)\right\}, \tag{1}$$

where $x_n(t)$ is the position of the nth vehicle at time t, $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ represents the headway between the $(n-1)$th and the nth vehicles, α is the sensitivity constant, τ is the reaction-time delay of drivers, which is different from the characteristic relaxation time $T = \alpha^{-1}$. The optimal velocity function $U(\Delta x_n(t))$is adopted the following form [13]

$$U(\Delta x_n) = 16.8[\tanh 0.0860(\Delta x_n - 25) + 0.913] \tag{2}$$

where $U(\cdot)$ is a monotonically increasing function and has an upper bound.

Using the expansion [31] for small τ, Eq. (1) is approximated by the following first-order acceleration equation

$$\tau\dot{a}_n(t) + a_n(t) = \alpha\left\{U(\Delta x_n(t)) - v_n(t)\right\}, \tag{3}$$

where $v_n(t)$ and $a_n(t)$ are the velocity and the acceleration of the nth vehicle at time t, respectively. Assuming that the leading vehicle runs at a fixed speed v_0, the steady-state uniform flow solution of Eq. (1) is given as

$$\left[v_n^*(t), y_n^*(t)\right]^T = \left[v_0, U^{-1}(v_0)\right]^T. \tag{4}$$

In order to derive its stability condition, the linearization of Eq. (3) around solution (4) leads to

$$\tau\frac{\mathrm{d}^3\bar{v}_n(t)}{\mathrm{d}t^3} + \frac{\mathrm{d}^2\bar{v}_n(t)}{\mathrm{d}t^2} + \alpha\frac{\mathrm{d}\bar{v}_n(t)}{\mathrm{d}t} + f\alpha\bar{v}_n(t) = f\alpha\bar{v}_{n-1}(t), \tag{5}$$

where $\bar{v}_n(t) = v_n(t) - v_0$ is a small velocity perturbation and $f = U'(y)\big|_{y=U^{-1}(v_0)}$. When $\tau = 0$, the stability criteria of Eqs. (1) and (3) is given as $\alpha < 2f$ [13]. When $\tau \neq 0$, the Laplace transformation is taken on both sides of Eq. (5)

$$Y_n(s) = G(s)Y_{n-1}(s), \tag{6}$$