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Dirac and the dispensability of mathematics

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Abstract

In this paper, I examine the role of the delta function in Dirac's formulation of quantum mechanics (QM), and I discuss, more generally, the role of mathematics in theory construction. It has been argued that mathematical theories play an *indispensable* role in physics, particularly in QM [Colyvan, M. (2001). *The indispensability of mathematics*. Oxford University Press: Oxford]. As I argue here, at least in the case of the delta function, Dirac was very clear about its *dispensability*. I first discuss the significance of the delta function in Dirac's work, and explore the strategy that he devised to overcome its use. I then argue that even if mathematical theories turned out to be indispensable, this wouldn't justify the commitment to the existence of mathematical entities. In fact, even in successful uses of mathematics, such as in Dirac's discovery of antimatter, there's no need to believe in the existence of the corresponding mathematical entities. An interesting picture about the application of mathematics emerges from a careful examination of Dirac's work.

Keywords: Dirac; Delta function; Indispensability argument; Quantum mechanics; Antimatter; Application of mathematics

1. Introduction

Among the various contributions made by Paul Dirac, one, in particular, is fascinating for the plethora of philosophical issues it raises: his introduction of the delta function in 1930 (see Dirac, 1958[1]). In a time when the physics community was deeply

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[1]The first edition of Dirac (1958) was published in 1930.

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concerned about establishing the equivalence between matrix and wave mechanics (see Muller, 1997), Dirac provided the first successful attempt to prove the equivalence result.[2] But Dirac's proposal relied, in an important way, on the delta function.

As is well known, the delta function has peculiar properties (among them, the fact that it's inconsistent!),[3] and so it's not surprising that Dirac would be suspicious about its use. (To avoid the use of this function, von Neumann would later introduce the Hilbert space formalism; see von Neumann, 1932.) Strictly speaking, the delta function is not even a function. And Dirac was, of course, aware of this. He called it an "improper function", since it doesn't have a definite value for each point in its domain (Dirac, 1958, p. 58). Moreover, as Dirac also points out, the delta function is ultimately dispensable. After all, it's "possible to rewrite the theory (i.e. quantum mechanics) in a form in which the improper functions appear all through only in integrands. One could then eliminate the improper functions altogether" (Dirac, 1958, p. 59).

In this paper, I examine the role of the delta function in Dirac's formulation of quantum mechanics (QM), which provides an interesting opportunity to examine, more generally, the role of mathematics in theory construction. It has been argued that mathematical theories play an *indispensable* role in physics, particularly in QM, given that it's not possible even to express the relevant physical principles without the use of mathematics. And given that mathematical entities are indispensable to our best theories of the world, the argument goes, we ought to believe in their existence (for a thorough defense of this view, see Colyvan, 2001). As I argue here, at least in the case of the delta function, Dirac was very clear about its *dispensability*.

After discussing the significance of the delta function in Dirac's work, and exploring the strategy that he devised to overcome its use, I examine the importance of this strategy to the use of mathematics in physics. I then argue that even if mathematical theories were indispensable, this wouldn't justify the commitment to the existence of mathematical entities. To illustrate this point, I examine an additional and particularly successful use of mathematics by Dirac: the one that eventually led to the discovery of antimatter. As we will see, even in this case, there's no reason to believe in the existence of the corresponding mathematical entities. An interesting new picture about the application of mathematics emerges from the careful examination of Dirac's work.

2. Dirac and the delta function

2.1. Introducing the delta function

Dirac's (1958) work on the foundations of QM is one in a series of exceptional treatises produced on the subject between the late 1920s and the early 1930s,

[2] Schrödinger attempted to prove the equivalence before, but as he acknowledged, only one direction of the equivalence was established (see Schrödinger, 1926, and Muller, 1997 for a discussion).

[3] For example, the delta function entails that differential operators are integral operators, but this is never the case (for an illuminating discussion of this point, see von Neumann, 1932, pp. 17–27; especially pp. 23–27).