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Stochastic resonance in a fractional harmonic oscillator subject to random mass and signal-modulated noise

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HIGHLIGHTS

- A fractional harmonic oscillator with random mass is investigated.
- The system-gain varies non-monotonically with noise intensity and driving force.
- The system-gain varies non-monotonically with the system coefficient.
- The system-gain varies non-monotonically with the fractional exponent.

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ABSTRACT

Stochastic resonance in a fractional harmonic oscillator with random mass and signal-modulated noise is investigated. Applying linear system theory and the characteristics of the noises, the analysis expression of the mean output-amplitude-gain (OAG) is obtained. It is shown that the OAG varies non-monotonically with the increase of the intensity of the multiplicative dichotomous noise, with the increase of the frequency of the driving force, as well as with the increase of the system frequency. In addition, the OAG is a non-monotonic function of the system friction coefficient, as a function of the viscous damping coefficient, as a function of the fractional exponent.

1. Introduction

The phenomenon of stochastic resonance (SR) observed in systems driven by random and periodic forces, on which the original work done by Benzi et al. [1,2] was in the context of modeling the switch of the Earth's climate between ice ages and periods of relative warmth. It is concluded that non-linearity, periodic and random forces are the essential ingredients for the occurrence of SR in nonlinear systems. On the other hand, behavior similar to SR has also been found in linear systems subjected to multiplicative noise [3,4].

For the case that the molecules surrounding a Brownian particle not only collide with it, but also adhere to the Brownian particle for some random time interval, the Brown motion can be described by an equation with adhesion. As a result of this adhesion, the mass of a Brownian particle becomes a random quantity. The phenomenon of particles of different sizes performing random collision and adhesion can be found in biological and chemical models [5,6], for an instance, a nano-mechanical resonator which randomly absorbs and desorbs molecules [6]. M. Gitterman considered a Brown motion with

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random mass, and analyzed the first moment and stability conditions and found the stochastic resonance phenomenon with the presence of a periodic force [7–10]. The stochastic resonance phenomenon in an under-damped linear harmonic oscillator with fluctuating mass [11,12] and fluctuating frequency [11] was also studied.

In the previous works on resonant behavior in fractional oscillators, the term with fractional derivative was considered as a damping force. It is suggested that the fractional derivative damping not only serves as the role of classical damping force but also contributes to the elastic restoring force [13,14]. On the other hand, viscous damping is also an import factor for the dynamic behavior of a harmonic oscillator. Chen L.C. et al. considered a Duffing oscillator with linear viscous damping and friction coefficient with fractional derivative [15]. Moreover, linear [3,4] or random viscous damping [16,17] is often investigated for second-order harmonic oscillators. However, to our knowledge, little attention has been paid on the effect of both the friction coefficient and viscous damping in a fractional oscillator. Thus in this work, we consider the stochastic resonant phenomenon in a fractional oscillator fluctuated by random mass with linear viscous damping coefficient and friction coefficient.

The structure of the paper is as follows. In Section 2, a mass-fluctuated fractional oscillator with linear viscous damping coefficient and fractional derivative is introduced. Exact formulas are found for the output amplitude gain. In Section 3, the SR phenomenon for the fractional oscillator is discussed. Finally some conclusions are drawn.

2. The fractional oscillator and its output amplitude gain

Considering a fractional oscillator coupled with a noisy environment and a power-law-type memory friction kernel described as the following stochastic differential equation

$$[1+\xi(t)]\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}+2r\frac{\mathrm{d}x(t)}{\mathrm{d}t}+\frac{k}{\Gamma(1-\alpha)}\int_0^t\frac{\dot{x}(u)}{(t-u)^\alpha}\mathrm{d}u+[\omega^2+\xi(t)]x(t)=\eta(t)\cdot A\cos(\Omega t), \tag{1}$$

where $x(t)$ is the oscillator displacement, r, k are the linear viscous damping and friction coefficients, respectively. α is the fractional exponent ($0<\alpha<1$), $\Gamma(.)$ is the gamma function. In the present work, $\xi(t)$ fluctuates both the mass and the frequency of the oscillator, which exists in the case of the fluctuation of the mass and frequency being from the same source. For example, in an RLC circuit, the environment temperature and humidity can simultaneously affect the value of the resistance and capacitor. $\xi(t)$ is modeled as a Markovian dichotomous noise, which consists of jumps between two values a and $-a$, $a>0$, with stationary probabilities $p(a)=p(-a)=1/2$. Its mean and variance are

$$\langle\xi(t)\rangle=0,\qquad\langle\xi(t)\xi(t+\tau)\rangle=a^2\mathrm{e}^{-\lambda|\tau|}=D\mathrm{e}^{-\lambda|\tau|}, \tag{2}$$

where λ is the correlation rate of the dichotomous noise, and D is its intensity. $\eta(t)$ is a zero mean signal-modulated noise, with coupling strength P with noise $\xi(t)$, i.e., $\langle\xi(t)\eta(t)\rangle=P$.

Averaging Eq. (1), applying the characteristics of the noises $\xi(t)$ and $\eta(t)$, one gets

$$\frac{\mathrm{d}^2\langle x\rangle}{\mathrm{d}t^2}+\left\langle\xi(t)\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right\rangle+2r\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}+\frac{k}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\dot{x}(u)\rangle}{(t-u)^\alpha}\mathrm{d}u+\omega^2\langle x\rangle+\langle\xi(t)x\rangle=0. \tag{3}$$

Multiplying both sides of Eq. (1) with $\xi(t)$, and then averaging, one obtains

$$\left\langle\xi(t)\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right\rangle+D\left\langle\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right\rangle+2r\left\langle\xi(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle+\frac{k\mathrm{e}^{-\lambda t}}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\xi(u)\dot{x}(u)\rangle\,\mathrm{e}^{\lambda u}}{(t-u)^\alpha}\mathrm{d}u+\omega^2\langle\xi(t)x\rangle+D\langle x(t)\rangle=PA\cos(\Omega t). \tag{4}$$

Using the well-known Shapiro–Loginov procedure [18], one gets

$$\frac{\mathrm{d}\langle\xi x\rangle}{\mathrm{d}t}=\left\langle\xi\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle-\lambda\langle\xi x\rangle. \tag{5}$$

Applying the characteristics of the fractional derivative and Shapiro–Loginov equation, Eqs. (3) and (4) can be respectively rewritten as

$$\frac{\mathrm{d}^2\langle x\rangle}{\mathrm{d}t^2}+\left(\lambda+\frac{\mathrm{d}}{\mathrm{d}t}\right)^2\langle\xi x\rangle+2r\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t}+\frac{k}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\dot{x}(u)\rangle}{(t-u)^\alpha}\mathrm{d}u+\omega^2\langle x\rangle+\langle\xi x(t)\rangle=0, \tag{6}$$

and

$$\left(\lambda+\frac{\mathrm{d}}{\mathrm{d}t}\right)^2\langle\xi x\rangle+D\left\langle\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right\rangle+2r\left\langle\xi(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\rangle+\frac{k\mathrm{e}^{-\lambda t}}{\Gamma(1-\alpha)}\int_0^t\frac{\langle\xi(u)\dot{x}(u)\rangle\,\mathrm{e}^{\lambda u}}{(t-u)^\alpha}\mathrm{d}u+\omega^2\langle\xi(t)x\rangle+D\langle x(t)\rangle=PA\cos(\Omega t). \tag{7}$$