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Boltzmann and Gibbs: An attempted reconciliation ☆

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Abstract

There are three levels of description in classical statistical mechanics, the microscopic/dynamic, the macroscopic/statistical and the thermodynamic. At one end there is a well-used concept of equilibrium in thermodynamics and at the other dynamic equilibrium does not exist in measure-preserving reversible dynamic systems. Statistical mechanics attempts to situate equilibrium at the macroscopic level in the Boltzmann approach and at the statistical level in the Gibbs approach. The aim of this work is to propose a reconciliation between these approaches and to do so we need to reconsider the concept of equilibrium. Our proposal is that the binary property of the system being or not being in equilibrium is replaced by a continuous property of *commonness*.

Keywords: Boltzmann; Gibbs; Approach to equilibrium; Ergodic decomposition

1. Introduction

One of the fundamental problems in the foundations of statistical mechanics is to give an explanation as to why 'equilibrium' statistical mechanics is so successful. That is to say, why the use of the standard Gibbsian methods 'reproduces'

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thermodynamic results. One offered explanation for this is the standard ergodic approach. As to whether this gives an acceptable explanation Van Lith (2001) offers the impression "that the *communis opinio* in the physics literature is that it does; in the philosophy literature that it doesn't". My impression agrees with hers. However, there is a further twist to the story. When confronted with the question of what is 'actually going on' in a gas of particles (say) when it is in equilibrium, or when it is coming to equilibrium, many physicists are quite prepared to desert the Gibbsian approach entirely and to embrace a Boltzmannian view (Ruelle, 1991; Lebowitz, 1993; Bricmont, 1995; Goldstein, 2001). In particular according to Goldstein:

> "Ludwig Boltzmann explained how irreversible macroscopic laws... originate in the time-reversible laws of microscopic physics. Boltzmann's analysis... is basically correct. The most famous criticisms of Boltzmann's later work on the subject have little merit. Most twentieth century innovations – such as the identification of the state of a physical system with a probability distribution ρ on phase space, of its thermodynamic entropy with the Gibbs entropy of ρ, and the invocation of the notions of ergodicity and mixing for the justification of statistical mechanics – are thoroughly misguided" (Goldstein, 2001, p. 39).

and Lebowitz:

> "Having results for typical microstates rather than averages is not just a mathematical nicety but at the heart of understanding the microscopic origin of observed macroscopic behaviour. We neither have nor do we need ensembles... . What we do need and can expect is typical behaviour" (Lebowitz, 1993, p. 38).

These assertions are reinforced by the opinion of Ruelle[1] that the Boltzmannian approach

> "is now generally accepted by physicists. ... There are some dissenting voices, such as that of Ilya Prigogine, but the disagreement is based on philosophical prejudice rather than physical evidence" (Ruelle, 1991, p. 113).

However, most work in equilibrium statistical mechanics uses the tools developed by Gibbs. Given a particular thermodynamic setup and microscopic model the appropriate probability distribution (microcanonical, canonical, grand-canonical, etc.) is chosen. The entropy is taken to be that of Gibbs and the holy grail for any investigation is an analytic form for the partition function; the notable successes being the solution of the zero-field two-dimensional Ising model by Onsager (1944), of the six-vertex model in 1967 by Lieb (see, Lieb & Wu, 1972) and of the eight-vertex model in 1972 by Baxter (see, Baxter, 1982). There have been many attempts to extend the Gibbs approach to non-equilibrium. As indicated above in the quote

[1] I think Ruelle rather overstates the case. In particular the rational subjectivist approach of Jaynes (1983) and the interventionist approach most recently argued for by Ridderbos and Redhead (1998) and Ridderbos (2002) would find some favour.