Studies in History and Philosophy of
Modern Physics 36 (2005) 137–163

Studies in History
and Philosophy
of Modern Physics

www.elsevier.com/locate/shpsb

Explaining quantum spontaneous symmetry breaking☆

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Received 27 April 2004; accepted 28 September 2004

Abstract

Two accounts of quantum symmetry breaking (SSB) in the algebraic approach are compared: the representational and the decompositional account. The latter account is argued to be superior for understanding quantum SSB. Two exactly solvable models are given as applications of our account: the Weiss–Heisenberg model for ferromagnetism and the BCS model for superconductivity. Finally, the decompositional account is shown to be more conducive to the causal explanation of quantum SSB.

Keywords: Spontaneous symmetry breaking; Idealization; Thermodynamic limit; Foundations of physics

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1. Introduction

The best known examples of spontaneous symmetry breaking (SSB) are found, many may think, in relativistic quantum field theory (QFT). For an up-to-date

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☆GGE acknowledges the hospitality of All Souls College, Oxford, during the Hilary term of 2004, and in particular the stimulating conversations with Jeremy Butterfield. We thank Jeremy for reading a draft of this paper and giving us detailed comments. We would also like to thank Laura Ruetsche for her comments on parts of the paper.

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doi:10.1016/j.shpsb.2004.12.003

account of the mathematical state of the art see, for instance, Ojima (2003) and the references therein. Then why, one may ask, do we choose to discuss SSB in non-relativistic quantum statistical mechanics (QSM)? While the problems of SSB in QFT may challenge physicists or mathematicians and fascinate philosophers of physics, they are not, as we shall argue, the best problems (i.e. offering the best models) for understanding the nature of quantum SSB. The tentative and controversial nature of some of the famous results of SSB in QFT is often an impediment to such an understanding, and the paucity of exactly solvable and experimentally realizable models does not help either. In contrast, SSBs in quantum thermo-systems with infinitely many degrees of freedom are well understood and have simple and realizable models. It is not at all accidental that when it comes to the discussion of SSB per se, authors in the QFT literature often resort to the analogous examples in QSM, examples such as ferromagnetism.

If so, one may wonder, why can we not search for the meaning of SSB in classical models, which would be an even simpler task? As shown in a detailed study (cf. Liu, 2003), classical SSB lacks several features that characterize *quantum* SSB. Therefore, part of the aim of this paper is to show how the essential features of SSB in QFT qua SSB manifest themselves clearly in systems of QSM, assuming that it is always preferable to study the simpler model, provided that all the essential features are captured, and none is left out.

Hence, in this paper we address the interpretative problems of explaining SSB in infinite quantum thermo-systems—the proper subjects of QSM. The main interpretative questions that are relevant to understanding how quantum SSB is understood include:

1. Why do quantum SSBs occur only in infinite systems? What justifies the use of such systems?
2. Why do quantum SSBs occur if, and only if, the symmetries in question are not unitarily implementable? What does this mean physically?
3. Why must the degenerate fundamental states[1] of an SSB system belong to unitarily inequivalent representations? What does this mean physically?
4. What is the physics of quantum SSB, in contrast to that of classical SSB?

To those readers who may worry that little philosophical interest can be generated from the above questions, we respond briefly as follows. (i) The nature of quantum SSB (markedly different from classical SSB) is itself of interest to metaphysics. An analogous case in this respect would be quantum measurement. (ii) Quantum SSB is closely related to the problem of quantum measurement. In a proposed solution to the problem (cf. Whitten-Wolfe & Emch, 1976), a measured quantum system triggers

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[1] The phrase, ‘fundamental states,’ denote the ‘lowest stable energy states’ in classical mechanics, the ‘equilibrium states’ in statistical physics, and the ‘vacuum states’ of quantum field theory. In the literature, ‘ground states’ are often used for such a purpose, but the term can be misleading. In this paper we use this phrase to mean the lowest stable energy states in all contexts except in QSM, where a system’s total energy is infinite, the role of such states are played by the KMS states.