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Traffic flow through multi-lane tollbooths on a toll highway

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ABSTRACT

We study the traffic states and queuing occurring in traffic flow on a toll highway with multi-lane tollgates. The traffic states change with increasing density and varying number of tollgates. When the manual-collection vehicles sort themselves into the tollgates, the queues occur just in front of the tollgates if the vehicular density is higher than a critical value. The queuing in front of tollgates is induced by the competition between the lane expansion and slowdown effects. When the lane expansion effect is superior to the slowdown effect, no queuing occurs. We derive the fundamental diagrams (current–density diagrams) for the traffic flow on the toll highway. The current saturates at the nearest tollgate at a low density and the saturation extends to the next-nearest tollgate with increasing density.

1. Introduction

Traffic flow is a self-driven many-particle system of strongly interacting vehicles [1–5]. Several physical models have been applied to the vehicular flow [6–37]. The dynamical phase transitions such the jamming transitions between distinct traffic states have been studied from the point of view of statistical physics and nonlinear dynamics.

Traffic jams are a typical signature of the complex behavior of traffic flow. Traffic jams are classified into two kinds of jams: (1) spontaneous jam (or phantom jam) which propagates backward as the stop and go wave and (2) stationary jam which is induced by slowdown or blockage at a section of the roadway. If the sensitivity of a driver is lower than a critical value, the spontaneous jam occurs. The jamming transition is very similar to the conventional phase transitions and critical phenomena [1,26]. When the sensitivity is higher than the critical value, the spontaneous jam does not appear, while the stationary jam induced by slowdown occurs [1,27].

In real traffic on toll roads, manual tollgates induce frequently the queuing just in front of traditional tollbooths because the delay occurs by collecting tolls manually (as cash). The tollgate works as a bottleneck. When the density is high, the jams or queues occur just before manual tollgates. In order to reduce the delay or congestion, more tollbooths have been introduced into the toll roads. The delay at a tollgate is reduced by increasing the tollgates. The travel time is reduced by the use of more tollgates.

In order to avoid the bottleneck effect of tollgates, more tollgates are set. The toll highway has the structure of lane expansion near the tollbooths. When vehicles enter into the multi-lane tollgates and if the tollgates are not congested, the driver goes straight toward the nearest tollgate. However, if the tollgates are congested, the vehicles sort themselves into not congested tollgates. The vehicular behavior changes with the number of tollgates. The queue occurs by competition between the lane expansion and slowdown effects. The occurrence of queues varies with the number of tollgates. It is necessary and important to know the traffic states and formation of queues. However, the traffic flow on a toll highway with multi-lane tollgates has been little studied by using modern traffic models.

In this paper, we present the dynamic model for the traffic flow of manual-collection vehicles on a toll highway with multi-lane tollgates. We apply the optimal velocity model to the multi-lane traffic flow on a toll highway with 1–5 tollgates.

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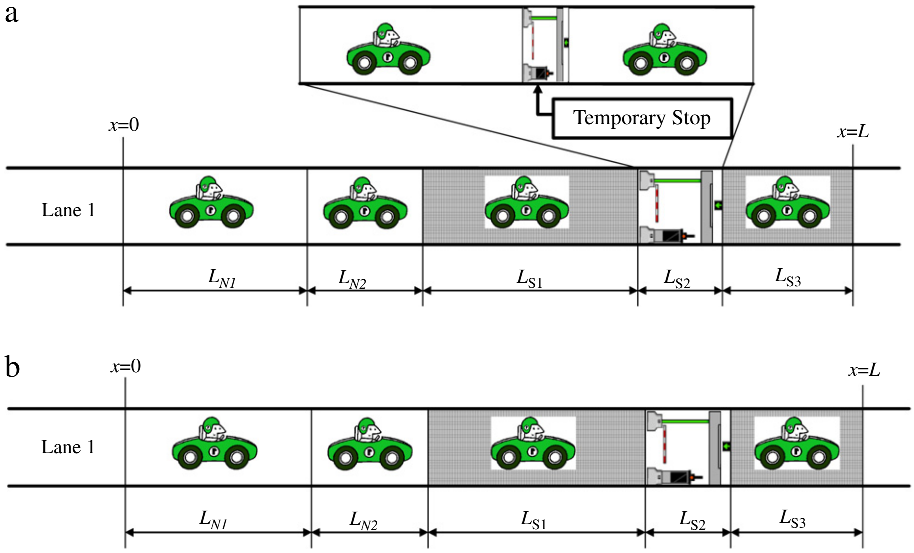

Fig. 1. (a) Schematic illustration of the traffic model for the single-lane highway with a tollgate. The toll highway is divided into 5 sections: N1, N2, S1, S2, and S3. Vehicles stop temporally at the tollgate within section S2. (b) Simplified model of (a). Section S2 including the tollgate is replaced by a slowdown section.

We study the traffic states and queues induced by the tollgates on the multi-lane highway. We clarify the dynamical states of traffic flow and the fundamental diagrams on the multi-lane toll highway.

2. Model

We consider the traffic situation of multi-lane toll highway. We introduce the multi-lane tollgates into a single-lane highway. When vehicles enter into the multi-lane tollgates, they slow down and expand to sort themselves into not congested tollgates. We mimic the traffic flow on the toll highway with multi-lane tollgates. We consider the traffic flow under the open boundary condition. Fig. 1(a) shows the schematic illustration of the traffic model for the single-lane highway with a tollgate. Vehicles enter into the toll highway at a finite density at $x = 0$. When vehicles approach to the tollbooth, they slow down, stop temporally at the tollgate with furthermore slowing down, and speed up again after they pass through the tollgate. If vehicles reach to end $x = L$ of toll highway, they are removed from the highway. We divide the toll highway into 5 sections: N1, N2, S1, S2, and S3. Lengths of their sections are $L_{N1}, L_{N2}, L_{S1}, L_{S2}, L_{S3}$ where total length L of the highway is $L = L_{N1} + L_{N2} + L_{S1} + L_{S2} + L_{S3}$. Vehicles move with normal speed at sections N1 and N2. At sections S1, S2, and S3, vehicles move with low speed. Furthermore, section S2 is divided into 3 subsections where section S2 includes the tollgate. Vehicles stop temporally at the tollgate within section S2. The model in Fig. 1(a) is simplified by that in Fig. 1(b). Here, section S2 including the tollgate is replaced by a slowdown section.

Fig. 2 shows the schematic illustration of the toll highway models with (a) two-lane, (b) three-lane, (c) four-lane, and (d) five-lane tollgates. In the toll highway with multi-lane tollgates, the single lane expands to multi-lanes if vehicles approach to the tollbooths. When vehicles approach to the tollgates, they slow down and pass through the tollgate at lowest speed. After vehicles pass through the tollbooth, they speed up. Vehicles move with low speed at sections S1 and S3. Vehicles pass through section S2 at lowest speed.

The merging section at the exit on the toll highway does not induce the congestion by the bottleneck effect because the bottleneck effect of tollgates is more effective than that at the exit. The bottleneck effect near the exit is not important because the length of road S3 is enough long and the bottleneck effect of tollgates is stronger than that of the exit.

We assume that vehicles are forced to slow down when they enter into sections of the slowdowns. When vehicles enter into normal section N2 and slowdown section S1, vehicles change the lane if the criteria of lane changing are satisfied. Lane changing is implemented as a sideways movement. We assume that the vehicular movement is divided into two parts: the one is the forward movement and the other is the sideways movement. We apply the optimal velocity model to the forward movement [1,28]. The optimal velocity model is described by the following equation of motion of vehicle i:

$$\frac{d^2 x_i}{dt^2} = a\left\{V(\Delta x_i) - \frac{dx_i}{dt}\right\}, \tag{1}$$

where $V(\Delta x_i)$ is the optimal velocity function, $x_i(t)$ is the position of vehicle i at time t, $\Delta x_i(t) (= x_{i+1}(t) - x_i(t))$ is the headway of vehicle i at time t, and a is the sensitivity (the inverse of the delay time) [1].