Physica A 431 (2015) 152–165

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Global exponential convergence of generalized chaotic systems with multiple time-varying and finite distributed delays

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HIGHLIGHTS

- We study the convergence of generalized chaotic systems with mixed delays.
- The bounds of the entire system are induced by the bounds of its partial variables.
- The delays are multiple time-varying and distributed delays.
- Some novel delay-dependent criteria ensuring convergence to a ball are obtained.
- We give some methods for calculating the maximum convergence rates.

ARTICLE INFO

Article history:
Received 16 August 2014
Received in revised form 6 February 2015
Available online 9 March 2015

Keywords:
Generalized chaotic systems
Global exponential convergence
Time-varying delays
Distributed delays
Partial variables
Inequality

ABSTRACT

Under some simple conditions, the convergence of a generalized chaotic system about its all variables is derived by only considering the convergence of its partial variables. Furthermore, based on some inequality techniques and employing the Lyapunov method, some novel sufficient criteria are derived to ensure the state variables of the discussed mixed delay system to converge, globally exponentially to a ball in the state space with a pre-specified convergence rate. Meanwhile, the ultimate bounds of the generalized chaotic system about its all variables are induced by the ultimate bounds of the system about its partial variables. Moreover, the maximum convergence rates about partial variables are also given. The methods are simple and valid for the convergence analysis of systems with time-varying and finite distributed delays. Here, the existence and uniqueness of the equilibrium point needs not to be considered. These simple conditions here are easy to be verified in engineering applications. Finally, some illustrated examples are given to show the effectiveness and usefulness of the results.

1. Introduction

Dynamical networks have been intensively studied [1–20] in the recent years and have been used in many applications such as optimization, control and image processing. In such applications, the key problem is to know the convergence properties of the designed neural network. However, during its design and hardware implementation, the convergence of a network may often be destroyed by its unavoidable uncertainty of models such as the signal transmission delays and external disturbance. Recently, it has been well recognized that constant or time-varying delays are often encountered in

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http://dx.doi.org/10.1016/j.physa.2015.03.001
0378-4371/

various neural networks [1–20], and the delays are often the sources of oscillations, instability and poor performance of the networks [21]. Therefore, for the necessity of applications, the dynamical characteristics of neural networks with time delays have become a subject of intense research activities.

Meanwhile, it is important to investigate the convergence of delayed neural networks (DNNs). Recently, some results concerning the convergence of different classes of neural networks have been widely discussed in Refs. [1–15], where Lyapunov stability of the unique equilibrium point of DNNs is mostly considered. Otherwise, in fact, when a neural network is designed to function as an associative memory, it is desired that the neural network should have multiple equilibrium points [22–25]. That is, the uniqueness of the equilibrium point may not be obtained. From a dynamical system point of view, globally stable networks in Lyapunov sense are monostable systems, which have a unique equilibrium attracting all trajectories asymptotically. In many other applications, however, monostable neural networks have been found to be computationally restrictive and multistable dynamics are essential to deal with the important neural computations desired. In these circumstances, networks are no longer globally stable and more appropriate notions of stability are need to deal with multistable systems, especially, such as the Cohen–Grossberg neural network, when applications are taken into account in biology, it is necessary and important to deal with multistable properties. Hence, the convergence is considered as the basis of the boundedness of solutions and the existence of global attractive sets [26–38]. In particular, the convergence plays an important role in chaotic systems [38] and multiagent networks [39,40]. About convergence, more results in the theory and application of dynamical systems refer to Refs. [1–24,26–38,41].

Based on the above discussion, our objective in this paper is to study the global exponential convergence (GEC) in Lagrange sense for a class of generalized chaotic networks with mixed delayed, not discussing the existence and uniqueness of the equilibrium point. It is believed that the results are significant and useful for the design and applications of the chaotic neural networks with mixed delays. It has been also noticed that some papers discussed the convergence (stability) in Lyapunov sense based on the Lyapunov function approach, the obtained conditions are either difficult to be verified or reformulated to be in a conservative form such as a linear matrix inequality (LMI) [6,8]. Furthermore, in general, the structure of a network is partially known or even completely unknown, which causes that it is very difficult to achieve the expected network properties. Therefore, in this paper, by applying some inequality technique, some simple delay dependent criteria are derived to ensure the globally exponential convergence (in Lagrange sense) of generalized delayed chaotic networks by only considering the convergence (in Lagrange sense) of its partial variables. Meanwhile, the ultimate bounds of the generalized chaotic system about its all variables are induced by the ultimate bounds of the system about its partial variables. Moreover, the maximum convergence rates with respect to partial variables are also given.

The rest of the paper is organized as follows: a generalized chaotic network model with mixed delays is presented and some preliminaries are introduced in Section 2. In Section 3, the exponential convergence of mixed delays chaotic network is discussed. In Section 4, some examples are given to illustrate the effectiveness of our results. Finally, the concluding remarks are drawn in Section 5.

2. Preliminaries

Consider the following chaotic systems with time-varying and finite distributed delays:

$$\begin{cases} \dot{x}_i(t) = \alpha_i(x_i(t))\Big[-\beta_i(x_i(t)) - \gamma\sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_j(t))) \\ \qquad + \sum_{j=1}^{n} d_{ij}\int_{t-p_j(t)}^{t} g_j(x_j(s))\,\mathrm{d}s + I_i(t)\Big], \quad i = 1, 2, \ldots, n, \\ \dot{y}_j(t) = k_j(x(t)) + \sum_{r=1}^{m} c_{jr}y_r(t) + J_j(t), \quad j = 1, 2, \ldots, m, \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $y(t) = (y_1(t), y_2(t), \ldots, y_m(t))^T$ are the state vectors of system (1). $\alpha_i(x_i(t)) > 0$ and $\beta_i(x_i(t)) > 0$ for $i \in \Lambda = \{1, 2, \ldots, n\}$. Functions $f_i(\cdot)$ and $g_i(\cdot)$: $R \longrightarrow R$ are continuous, $\gamma > 0$ is a constant, $k_j(\cdot) \in C[R^n, R]$ for $j \in \Upsilon = \{1, 2, \ldots, m\}$. $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$ are the connection weight matrices, the delayed weight matrices and the distributed delayed connection weight matrices, respectively. $C = (c_{ij})_{m\times m}$ is a real matrix, which denotes the strength of neuron interconnections. The external input vector functions $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^T$, $J(t) = (J_1(t), J_2(t), \ldots, J_m(t))^T$ are assumed to be bounded and the time-varying delays $\tau_i(t)$, $p_i(t)$ are all nonnegative. We define $I_i = \sup_{t\geq 0} |I_i(t)|$, $J_j = \sup_{t\geq 0} |J_j(t)|$, $I = (I_1, I_2, \ldots, I_n)^T$, $J = (J_1, J_2, \ldots, J_m)^T$.

Remark 1. System (1) contains some other networks given in Refs. [1,3,4,6,8,17–20] as special cases. For examples, when the matrices $C = 0$ and $k(x(t)) = J(t) = 0$, system (1) reduces to the chaotic Cohen–Grossberg neural networks in Refs. [3,4,8], and when $C = 0$, $k_j(x(t)) = J_j(t) = 0$ and $D = 0$, system (1) simplifies to the chaotic Cohen–Grossberg neural network in Refs. [17,20]. Meanwhile, if $\alpha_i(x_i(t)) = 1$, $f_i(x_i(t)) = x_i(t)$, $D = 0$ with $I(t) = 0$ and $J(t) = 0$, then system (1) reduces to the delayed chaotic network in Ref. [19].