Physica A 458 (2016) 303–312

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Correlated biased random walk with latency in one and two dimensions: Asserting patterned and unpredictable movement

E. Rodriguez-Horta [a,b], E. Estevez-Rams [a,b,*], R. Lora-Serrano [c], B. Aragón Fernández [d]

[a] Facultad de Física, University of Havana, San Lazaro y L. CP 10400, La Habana, Cuba
[b] Instituto de Ciencias y Tecnología de Materiales, University of Havana (IMRE), San Lazaro y L. CP 10400, La Habana, Cuba
[c] Universidade Federal de Uberlandia, AV. Joao Naves de Avila, 2121- Campus Santa Monica, CEP 38408-144, Minas Gerais, Brazil
[d] Universidad de las Ciencias Informáticas (UCI), Carretera a San Antonio, Boyeros, La Habana, Cuba

HIGHLIGHTS

- The balance between irreducible random movement and structured movement is reported for the correlated biased random walk model with latency (CBRWL).
- The CBRWL in two dimensions is constructed from the orthogonal extension of the one dimensional model.
- The relation between drift velocity and tortuosity as a function of entropy density and excess entropy.
- Entropic measures allows to characterize the dynamics independently of the particular control variables.

ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form 3 March 2016
Available online 12 April 2016

Keywords:
Random walk
Computational mechanics
SHANNON entropy

ABSTRACT

The correlated biased random walk with latency in one and two dimensions is discussed with regard to the portion of irreducible random movement and structured movement. It is shown how a quantitative analysis can be carried out by using computational mechanics. The stochastic matrix for both dynamics are reported. Latency introduces new states in the finite state machine description of the system in both dimensions, allowing for a full nearest neighbor coordination in the two dimensional case. Complexity analysis is used to characterize the movement, independently of the set of control parameters, making it suitable for the discussion of other random walk models. The complexity map of the system dynamics is reported for the two dimensional case.

Dynamical systems can be seen as generating and storing information. In this sense, they can be described as symbols generating computational machines. To know the extent that a given dynamical system is capable of such computing capacity can be important if, for example, one intends to practically tune the control parameters of the system to take advantage of its computing ability. Starting from Brownian motions, the random walk (RW) has been a much used model in statistical physics and related fields to simulate the behavior of different physical systems [1,2]. Its ubiquitous nature, has made it useful in the analysis of a broad number of situations beyond physics such as, biology, where it has been used to model the movement from micro-organisms and insects to mammals [3] or, the diffusion of charges in solids [4].

* Corresponding author at: Facultad de Física, University of Havana, San Lazaro y L. CP 10400, La Habana, Cuba.
E-mail address: estevez@imre.uh.cu (E. Estevez-Rams).

http://dx.doi.org/10.1016/j.physa.2016.03.017
0378-4371/

The classical discrete RW involves a stochastic movement of a given length in some arbitrary direction in d-dimensional space. In what follows we will consider that the length of the step is constant over the whole walking process, and in consequence, a fixed time is taken for each step. We will also consider a lattice space, which implies a discrete space over which the walker movement is performed [5]. In its more simple case [5,6], at a given time, the choice of movement direction is a random variable following a given probability distribution with no memory whatsoever of the previous movements. Enlarging this model can be made in several ways. One choice is to introduce biased as a favored direction in space, while correlation is considered when the choice of movement takes into account the previous movements of the walker (the process has memory) [7]. Biased can be taken as a consequence of some driving force (e.g. a particular odor for a predator, an external field for a charge moving inside a solid, etc.) The most simple form of correlation, usually called persistence, is to consider, when choosing the next movement, the last step of the walker. Persistence then introduces a preference, that the walker follows the previous movement as some kind of inertial effect [8].

A further variable in the movement can be introduced if the possibility that, at a given time, the walker decides to stay at rest allowed [3]. The resulting RW will be said to have latency. Latency could be justified in a number of ways. One could consider it to be a transitional state between change of movement direction, due to the impossibility of instantaneous movement change. Yet, we will spare any rationale leading to latency and just consider it for its worth in enhancing the possibilities of RW models. In particular, it will be shown that introducing latency in one dimensional RW, allows to go to a two dimensional lattice RW in a simple manner while keeping full coordination of the movement.

In the analysis of RW, it has been usual to be interested in the statistical properties of the movement, such as the probability distribution over distance as a function of time, from where the diffusion coefficient can be calculated. Other questions have been the probability that the walker will return to a given point in space; the speed of the walker; the probability of escaping the origin; among others [5,6,9]. Closely related to these questions, is to assert the balance between unpredictable and structured movement in the RW, as a consequence of the competing effect of the control parameters. This analysis can be carried out by coding the RW as a symbol producing system and then quantify the production of patterns and irreducible randomness in the symbolic generator process. Viewed from this perspective, the RW can be considered as a computational machine that sequentially generates an infinite string of symbols with certain degree of predictability and certain degree of noise. Structure is then associated to the relation between some magnitude characterizing the predictable behavior and some magnitude characterizing the random part [10].

Computational mechanics is an approach to complexity based on reconstruction of the optimal computational machine, termed ϵ-machine, within a hierarchy of machines that allows to discover the nature of patterns and to quantify them [11,10]. It is rooted in information theory concepts, and has found applications in several areas [12–14]. Its use in statistical mechanics has allowed to define and calculate magnitudes that complement thermodynamic quantities. Optimality is the balance of having the best predictive power with less number of resources as reflected by the number of internal states [15].

For an ϵ-machine, the statistical complexity C_μ is defined as the Shannon entropy over the probability of the causal states. A causal state is taken in its more general meaning, as a set of pasts that determines (probabilistically) equivalent futures [16]. C_μ is a measure of the amount of knowledge or memory needed for optimal prediction. Another magnitude is the entropy density, which measures the irreducible randomness of the system and is defined as the asymptotic value of the Shannon block entropy over the generated sequence of symbols divided by the block length [17]. And finally, the excess entropy, defined as the mutual information between past and future, which, in the case of a first order Markov process, can be calculated by subtracting the entropy density from the statistical complexity. Excess entropy is the amount of memory needed to make optimal predictions without taking into account the irreducible randomness [18].

In this article we will be interested in quantifying the unpredictable and predictable movement in a biased correlated RW with latency (CBRWL) in both one- and two-dimensions.

The paper is organized as follows: in Section 1, notions of computational mechanics are introduced for completeness. In this section the mathematical definition of statistical entropy, entropy density and excess entropy are introduced. Section 2 describes the one dimensional RW as a symbol generating process. In Section 3 the two dimensional case is discussed as an extension of the one dimensional walk in two orthogonal directions, this is followed in Section 4 by discussion and conclusions.

1. Casual states, statistical complexity, entropy density and excess entropy

Computational mechanics relies on the concept of causal states [16]. Consider a process whose output is a bi-infinite string or sequence, the characters of which are drawn from an alphabet Σ. Consider, for any particular realization of the process, the output string ξ and partition the string in two half $\xi^- \equiv \xi(-\infty, -1)$ and $\xi^+ \equiv \xi(0, \infty)$ which are called past and future, respectively. If the output strings are considered to be drawn from a (in general unknown) distribution, then two ξ^- and ξ'^- that give the same probability $Pr(\xi^+|\xi^-) = Pr(\xi^+|\xi'^-)$ for all possible futures ξ^+, are said to belong to the same causal state C_p that it is written $\xi^- \sim \xi'^-(\xi^-,\ \xi'^- \in C_p)$.

The partition of the set of possible pasts (denoted by Ξ^-), in classes of causal states, is an equivalence relation complying with the transitivity condition (if $\xi_i^- \sim \xi_j^-$ and $\xi_j^- \sim \xi_k^-$ then $\xi_i^- \sim \xi_k^-$), symmetry (if $\xi_i^- \sim \xi_j^-$ then $\xi_j^- \sim \xi_i^-$) and reflectivity ($\xi_i^- \sim \xi_i^-$). The set of causal state (denoted by $\mathcal{C}$ with cardinality $|\mathcal{C}|$) uniquely determines the future of a sequence. Then, function ϵ can be defined over ξ^-, $\epsilon : \Xi^- \to \mathcal{C}$ which relates ξ^- with its causal states C,

$$\epsilon(\xi^-) \equiv C = \{\xi'^-|Pr(\xi^+|\xi^-) = Pr(\xi^+|\xi'^-)\ \forall \xi^+\}.$$