Physica A 458 (2016) 329–341

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Critical Casimir effect in the Ising strips with standard normal and ordinary boundary conditions and the grain boundary

Z. Borjan

University of Belgrade, Faculty of Physics, P.O.Box 44, 11001 Belgrade, Serbia

HIGHLIGHTS

- Critical Casimir effect in Ising strips with internal grain boundary is considered.
- Casimir amplitudes are derived as functions of the grain boundary strength.
- Structure of short-distance expansions near the grain boundary is studied in detail.

ARTICLE INFO

Article history:
Received 13 August 2015
Received in revised form 18 February 2016
Available online 20 April 2016

Keywords:
Casimir effect
Ising strip
Grain boundary
Distant-wall corrections

ABSTRACT

We consider critical Casimir force in the Ising strips with boundary conditions defined by standard *normal* and *ordinary* surface universality classes containing also the internal grain boundary. Using exact variational approach of Mikheev and Fisher we have elaborated on behaviors of Casimir amplitudes $\Delta_{++}(g)$, $\Delta_{OO}(g)$ and $\Delta_{+O}(g)$, corresponding to normal–normal, ordinary–ordinary and mixed normal–ordinary boundary conditions, respectively, with g as a strength of the grain boundary. Closed analytic results describe Casimir amplitudes $\Delta_{++}(g)$ and $\Delta_{OO}(g)$ as continuous functions of the grain boundary's strength g, changing the character of the Casimir force from repulsive to attractive and vice versa for certain domains of g. Present results reveal a new type of symmetry between Casimir amplitudes $\Delta_{++}(g)$ and $\Delta_{OO}(g)$. Unexpectedly simple constant result for the Casimir amplitude $\Delta_{+O}(g) = \frac{\pi}{12}$ we have comprehensively interpreted in terms of equilibrium states of the present Ising strip as a complex interacting system comprising two sub-systems. Short-distance expansions of energy density profiles in the *vicinity of the grain boundary* reveal new distant-wall correction amplitudes that we examined in detail. Analogy of present considerations with earlier more usual short-distance expansions near one of the (N), (O) and (SB) boundaries, as well as close to surfaces with variable boundary conditions refers to the set of scaling dimensions appearing in the present calculations but also to the discovery of the de Gennes–Fisher distant wall correction amplitudes.

1. Introduction

One of the most renowned results in electrodynamics refers to the discovery of a long-range force between two perfectly conducting plates in vacuum at a distance L much smaller than their lateral extension [1]. It was found out that an interaction between the plates is conveyed by fluctuations of the electromagnetic field in vacuum. Known in the literature as the Casimir force, it arises due to constrained frequency spectrum of fluctuations imposed by plates playing the role of boundaries. The zero-point energy of the confined electromagnetic field becomes L-dependent while strength and nature (attractive or repulsive) of the Casimir force are also affected by the shape and type of boundaries.

E-mail address: zborjan@ff.bg.ac.rs.

http://dx.doi.org/10.1016/j.physa.2016.04.002

However, the Casimir effect is not a phenomenon exclusively characterizing the electromagnetic field. All confined physical systems featured by fluctuating quantities manifest long-range forces. Various boundaries in such systems enforce constraints on fluctuation spectra making the ground state energy or the free energy size and shape dependent. For this reason the Casimir effect is studied in several areas of physics such as electrodynamics, condensed matter physics, statistical physics [2–5], quantum chromodynamics and cosmology.

The order parameter of a thermodynamic system of the finite size L near the critical point fluctuates at distances set out by the correlation length ξ. Near the bulk critical point the correlation length becomes comparable with L. Specific boundary conditions are embodied in the behavior of the order parameter at confining surfaces. Due to confinement the study of the thermodynamic Casimir effect is based on the finite-size scaling theory [6–9]. The Casimir force in finite-size statistical systems with boundary conditions (ab) on the constituent surfaces is defined by

$$F^{(ab)}_{\mathrm{L,Cas}} = -\frac{\partial f^{(ab)}_{\mathrm{L,ex}}(T)}{\partial L}, \tag{1}$$

where $f^{(ab)}_{\mathrm{L,ex}}$ is the excess free energy $f^{(ab)}_{\mathrm{L,ex}} := f^{(ab)}_{\mathrm{L}} - L f_{\mathrm{bulk}}(T)$, with $f^{(ab)}_{\mathrm{L}}$ as the full free energy per unit area and per $k_B T$ while $f_{\mathrm{bulk}}(T)$ is the free energy density of unbounded system.

At the bulk critical point T_c the total free energy density of a d-dimensional critical system defined by two parallel plates with boundary conditions (a) and (b) respectively, at the distance L decomposes into a sum [2]:

$$f^{(ab)}_{\mathrm{L}} \simeq f_{\mathrm{bulk}}(T_c) + f^{(a)}_{s}(T_c) + f^{(b)}_{s}(T_c) + L^{-(d-1)}\Delta_{ab} + \cdots, \tag{2}$$

as the area A of plates $A \to \infty$ and $L \to \infty$. In the last equation $f^{(a)}_{s}$ and $f^{(b)}_{s}$ denote surface free energies of the opposite surfaces and Δ_{ab} is the Casimir amplitude which determines the critical $(T = T_c)$ Casimir force. According to the definition (1) the last term in (2) gives rise to the Casimir force

$$F^{(ab)}_{\mathrm{L,Cas}} = (d-1)\Delta_{ab} L^{-d}. \tag{3}$$

The finite-size term $L^{-(d-1)}\Delta_{ab}$ of (2) stems from scale invariance of the free energy [10]. Casimir amplitudes Δ_{ab} are *universal* [2–5,11] depending on the space dimensionality, the bulk universality class, the surface universality classes (ab) as well as on the geometry involved. For the Ising-like systems the free energy (2) may also contain interface free energy $\sigma^{(ab)}(T)$ besides the two L-independent terms $f^{(a)}_{s}$ and $f^{(b)}_{s}$. Eq. (2) is relevant for $O(n)$-symmetric models $(n \geq 1)$. These are, for example, one-component fluids at the liquid–vapor critical point, binary fluids at the consolute point $(n = 1)$, liquid ^{4}He at the λ transition point $(n = 2)$ etc.

Close to the bulk critical temperature T_c the Casimir force is long-ranged as (3) shows. A significant amount of evidence [2–5] suggests that the Casimir force is attractive $\Delta_{aa} < 0$ for the like boundary conditions at opposing surfaces. Much attention was devoted in the past to the study of the Casimir effect in the Ising-like systems for standard surface universality classes [2–5].

The surface with short-ranged interactions can undergo its own phase transition only if its dimensionality $d \geq 2$. Then there are three possibilities for surface and bulk phase transitions in the semi-infinite Ising system without external fields [12]. If the surface does not spontaneously order while the bulk transition already took place, the spontaneous bulk magnetization will force ferromagnetic ordering of the surface. This is the *ordinary* (O) transition. In case of the *extraordinary* (E) transition the surface spontaneously orders *before* the bulk phase transition occurs. The third possibility is that the surface and bulk transition to the ferromagnetic state take place simultaneously at the same critical temperature. This is the *surface-bulk* (SB) transition. The (SB) transition is a multicritical point where the lines of surface (S), (O) and (E) transitions meet. The SB transition rarely exists in the two-dimensional systems. The two-dimensional $O(n)$ model has the (SB) transition only for $n < 1$, therefore it is absent in the semi-infinite Ising model $(n = 1)$.

It is generally accepted that (E) transition, characterized as the spontaneously ordered surface, is equivalent to the surface ordered externally by the surface magnetic field. This is then called the *normal* (N) transition. Surface universality classes (O) and (SB) are refereed to as *symmetry-conserving* due to the absence of the surface order. On the other hand, (E) and (N) phase transitions break the $O(n)$ symmetry, therefore they are specified as the *symmetry-breaking*.

The Casimir effect has been most exhaustively scrutinized in the Ising-like systems. Exact results on Casimir amplitudes Δ_{ab} for the two-dimensional Ising model exist for strips under periodic (p), antiperiodic (a), ordinary (O), symmetric $(++)$ and antisymmetric $(+-)$ boundary conditions [9,13–15]. Boundary conditions $(++)$ denote the (N) surface universality class on the opposite surfaces with infinite magnetic fields of the same sign $h_i = +\infty$ $(i = 1, 2)$, while $(+-)$ assumes imposed infinite fields of opposite signs $h_1 = -h_2 = +\infty$. The universality class of the critical system in two dimensions is determined by the so called central charge c. Quite general studies imply that Casimir amplitudes are the *same* for $(++)$ and (OO) boundary conditions [13,16]:

$$\Delta_{++} = \Delta_{OO} = -\frac{\pi}{24}c. \tag{4}$$

The Ising universality class corresponds to $c = 1/2$. Due to the absence of (SB) fixed point for the two-dimensional Ising-like systems there are no estimates for Casimir amplitudes $\Delta_{+,SB}$, $\Delta_{O,SB}$ or $\Delta_{SB,SB}$.