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Betting on dynamics

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HIGHLIGHTS

- We first consider a game whose results represent the type of dynamics that the system exhibits.
- By introducing a second game, we show that different games can be defined to focus on different aspects of the physics of a system.
- Different winning strategies and Nash equilibrium points, related to distinct physical properties, emerge in each game.
- Special emphasis is made on Nash points, which add a new conceptual perspective, to analyze a physical system.

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ABSTRACT

A physical system in game-theoretic terms is represented in this work. We first consider an abstract game between a classical and a quantum player, whose results represent the two different kinds of non-linear dynamics that the system exhibits. The existence of winning strategies shows that the dynamics can be fully determined through a game theory analysis. Moreover by considering a second game, we show that different games can be defined to focus on different aspects of the physics of a system. Different winning strategies and Nash equilibrium points, emerge in each game. These games theory tools are related to physical properties, so distinct physical information is obtained for each game. In this work, special emphasis is made on Nash points, which add a new conceptual perspective, to analyze a physical system.

1. Introduction

The idea of extending game theory concepts [1] to the quantum field [2] has received considerable attention in the recent past. Classical game theory is a well understood mathematical discipline with many applications in economy, psychology, and biology [3–6]. Consequently, "quantizing" it has provided refreshing insights. Add to this the fact that several physics' problems can be fruitfully regarded as games [2,7–24]. We will try here to express physical properties in the language of games theory. Our attention will be concentrated on the semiclassical instance, that has a long and distinguished history.

In Refs. [25,26] we toyed with the idea de expressing physical model in game-theory parlance by considering the interaction between matter and a single-mode of an electromagnetic field. The associated game admits both classical and quantal players and strategies are determined by the initial conditions of the pertinent dynamical system. More specifically, in Ref. [25] we considered states in the way Meyer does [2] with regard to quantum coins. In Ref. [26], instead, we looked at the same physical scenario but considering orthogonal (distinguishable) initial states. Remarkably enough, it will be seen that this seemingly technical detail profoundly affects the whole picture.

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In Refs. [25,26], is defined an *intuitive* game with two options related to a particular aspect of the system: the populations of each level either increase or decrease. Here we are interested in: (1) Determining if it is possible, for practical purposes, to study general properties of a system through a game; and (2) Studying the implications of considering various games for the same system. Because we are considering a physical system, we expect that different games will emphasize different physical properties. A question then arises: Is there a relationship between the Nash Points of each game? In addition, it is important to find out to what physical features of the system these points are associated. To satisfy objectives (1) and (2), we will introduce two totally abstract games where the players will bet on the dynamics of the system, which can be classified by its behavior for $t \to \infty$ into two very different types.

2. Games: brief review of basic notions

In a game we deal with

- a finite set of N players plus a set of pure strategies s_i ($i = 1, 2, \ldots, N$) available to those players.
- Pure strategies providing a complete definition of how a player will play. In particular, they determine the moves a player will make for any situation she could face.
- A set of payoffs for each combination of strategies. Payoffs are numbers which represent the motivations of players. Payoffs may represent profit, quantity, utility, or other continuous measures (cardinal payoffs), or may simply rank the desirability of outcomes (ordinal payoffs). In all cases, the payoffs must reflect the motivations of the particular player.

Games admit as well *mixed strategies*, that are combinations of probability assignments to each available pure strategy. If $s_i = \{s_{i1}, \ldots, s_{ik}\}$, a mixed strategy for player i is a probability distribution $p_i = (p_{i1}, \ldots, p_{ik})$, where $p_{i1} + \cdots + p_{ik} = 1$. The payoff P_i is here replaced by an average payoff $\langle P_i \rangle$.

Nash equilibrium. Suppose that the theory makes a unique prediction about the strategy each player will choose. In order for this prediction to be correct, it is necessary that each player be willing to choose the strategy predicted by the theory. Thus, each player's predicted strategy must be that player's best response to the predicted strategies of the other players. Such a prediction could be called strategically stable, because no single player would want to deviate from his predicted strategy. This situation is called a Nash equilibrium. The mixed strategies $(p_1^*, \ldots, p_N^*)$ attain Nash equilibrium if, for each player i, p_i^* is player i's best response to the mixed strategies specified for the $N - 1$ remaining players, $(p_1^*, \ldots, p_{i-1}^*, p_{i+1}^*, \ldots, p_N^*)$. Thus,

$$\langle P_i \rangle (p_1^*, \ldots, p_{i-1}^*, p_i^*, p_{i+1}^*, \ldots, p_N^*) \geq \langle P_i \rangle (p_1^*, \ldots, p_{i-1}^*, p_i, p_{i+1}^*, \ldots, p_N^*), \tag{1}$$

for every feasible strategy p_i.

3. Quantum and semi-classical games

In *quantum games* expected payoffs will be calculated now via a density matrix ρ [25,26],

$$Tr(\rho P_i), \tag{2}$$

where P_i standing for a convenient "payoff" operator associated to each player (quantum and classical) and ρ in general a mixed density matrix [25,26]

$$\rho = \sum_j |Q_j\rangle\, p_j \langle Q_j|, \tag{3}$$

where $\sum_j p_j = 1$. We will call "classical" those players who choose probabilities p_j (mixed strategies). "Quantum players", instead, select quantum states $|Q_j\rangle$ as their strategies. The players need not be human beings. They are basically sets de strategies and payoffs. However, in this context, any experimentalist can be viewed as a classical player.

The density matrix (3) may represent agame played (i) by classical players if we keep fixed the $|Q_j\rangle$-states, (ii) between quantum players if $p_1 = 1$ and the remaining p_j vanish, and (iii) between both classical and quantum players. If there are several players, the probabilities p_j are expressed as products (of probabilities) and the states $|Q_j\rangle$ as tensor products of quantum states.

We consider the illustrative instance in which a classical C-player and a quantum Q-one play with two different strategies: a mixed one for player C and a quantum strategy for player Q. Matrix (3) expresses a situation in which *C-players* support with probability p_j (o reject with probability $1 - p_j$) the $|Q_j\rangle$-strategy.

We introduce next a pair of one-boson games that involves distinguishable, i.e., orthogonal states and overlapping ones.

4. A two level system

We consider now the following two-levels boson Hamiltonian [27–30]

$$H = E_1 N_1 + E_2 N_2 + \frac{\omega}{2}({P_X}^2 + X^2) + \gamma X\, (a_1^\dagger a_2 + a_2^\dagger a_1), \tag{4}$$

that represents matter interacting with a single-mode of an electromagnetic field within a cavity. One has $N_1 = a_1^\dagger a_1$ and $N_2 = a_2^\dagger a_2$, the population operators corresponding to the levels one and two, respectively, and we assume $E_2 > E_1$.