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The application of the fractional exclusion statistics to the BCS theory—A redefinition of the quasiparticle energies

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HIGHLIGHTS

- The quasiparticle system in the BCS model is shown to form a Fermi liquid (FL).
- The BCS FL is transformed into an ideal gas by redefining the quasiparticles.
- The new quasiparticles obey fractional exclusion statistics (FES).
- We calculate the FES parameters and verify the consistency of the model.
- A repulsive interaction between quasiparticles stabilizes the BCS condensate.

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ABSTRACT

The effective energy of a superconductor $E_{eff}(T)$ at temperature T is defined as the difference between the total energy at temperature T and the total energy at 0 K. We call the energy of the condensate, $\mathcal{E}_c$, the difference between E_{eff} and the sum of the quasiparticle energies E_{qp}. $\mathcal{E}_c$, E_{qp}, as well as the BCS quasiparticle energy ϵ are positive and depend on the gap energy Δ, which, in turn, depends on the populations of the quasiparticle states (equivalently, they depend on T). So, from the energetic point of view, the superconductor is a Fermi liquid of interacting quasiparticles.

We show that the choice of quasiparticles is not unique, but there is an infinite range of possibilities. Some of these possibilities have been explored in the context of the fractional exclusion statistics (FES), which is a general method of describing interacting particle systems as ideal gases. We apply FES here and transform the Fermi liquid of BCS excitations into an ideal gas by redefining the quasiparticle energies. The new FES quasiparticles exhibit the same energy gap as the BCS quasiparticles, but a different DOS, which is finite at any quasiparticle energy.

We also discuss the effect of the remnant electron–electron interaction (electron–electron interaction beyond the BCS pairing model) and show that this can stabilize the BCS condensate, increasing the critical temperature.

1. Introduction

We divide the energy of a BCS superconductor [1] into three parts: the ground-state energy E_{gs}, the condensate energy $\mathcal{E}_c$, and the energy of the quasiparticles E_{qp}. E_{gs} is a constant and represents the total energy of the superconductor at

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temperature $T = 0$, E_{qp} is the sum of the excitations' quasiparticle energies, whereas the condensate energy is the difference $\mathcal{E}_c \equiv E - E_{gs} - E_{qp}$. $\mathcal{E}_c$ vanishes at $T = 0$ and increases monotonically with T, reaching its highest value at the critical temperature T_c, where the superconducting state disappears. Effectively, the energy of the system-after removing the constant term E_{gs}-is $E_{\rm eff} \equiv \mathcal{E}_c + E_{qp}$. Due to the fact that both, $\mathcal{E}_c$ and the BCS quasiparticle energies ϵ, depend on the populations of the quasiparticle states $\{n_\epsilon\}$, $E_{\rm eff}$ represents the energy of a Fermi liquid (FL) [2,3] and $\epsilon \equiv \partial E_{\rm eff}/\partial n_\epsilon$.

In the context of fractional exclusion statistics (FES) [4–6] it has been shown that the quasiparticle energies may be redefined (see e.g. Refs. [7–18]). There is an infinite range of possibilities in which one can redistribute the energy of the system among the quasiparticle states. Moreover, if the choice is made such that the total (or the effective) energy of the system is equal to the sum of the quasiparticle energies, then one obtains a description of the system in terms of an ideal FES gas [17]. All the choices of quasiparticle energies must lead to thermodynamically equivalent descriptions, in the sense that the populations of the quasiparticle states and all the macroscopic thermodynamic quantities should not depend on the chosen description [17,18]. We exemplify the procedure by redefining the quasiparticle in such a way that $E_{\rm eff}$ becomes the sum of the new quasiparticle energies $\tilde{\epsilon}$. This relation holds for any quasiparticle levels populations, so the system obtained is an ideal gas. In our example, the quasiparticle energy spectrum exhibits the same energy gap Δ as that of the BCS quasiparticles, but the density of states (DOS) $\tilde{\sigma}(\tilde{\epsilon})$ is finite over the whole spectrum (including at $\tilde{\epsilon} = \Delta$).

We also extend the BCS model by including an extra interaction between the electrons as a perturbation to the initial pairing Hamiltonian. This leads to an interaction term between the quasiparticles which modifies the energy gap and the quasiparticle energies. The gap equation cannot be satisfied anymore for $\Delta = 0$ at any temperature, so, in the first order of perturbation, the extra interaction does not allow the superconducting phase to be destroyed.

The paper is organized as follows. In the next subsection we introduce the notations and the basic concepts of the BCS theory. We shall use mainly the notations of Ref. [19] which are somewhat different from the notations of Ref. [1]. Then, in Section 2, we write the effective energy of the system as the energy of a Fermi liquid (FL), with the BCS quasiparticle energies equal to the Landau's quasiparticle energy of the FL. The FES description is presented in Section 3, where we introduce the FES quasiparticle energies, the FES parameters, and we write the FES equations for the population. We also show that the FES and FL descriptions are physically equivalent. In Section 4 we extend the BCS model by introducing the interaction between the quasiparticles. In Section 5 we present the conclusions.

1.1. The basics of the theory of superconductivity

Let us specify notations and the basic ideas of the BCS theory, following mainly Refs. [19,1]. We denote the single-particle states of the electrons in the superconductor by $|\mathbf{k}, s\rangle$ and its time reversed state by $|-\mathbf{k}, -s\rangle$; s is the spin and $\mathbf{k}$ represents the rest of single-particle quantum numbers that specify the state-concretely, we shall consider that $\mathbf{k}$ is the free electron wavevector. The electrons creation and annihilation operators are $c^\dagger_{\mathbf{k},s}$ and $c_{\mathbf{k},s}$, respectively, and the BCS pairing Hamiltonian is

$$\mathcal{H}_{BCS} = \sum_{\mathbf{k}\sigma} \epsilon^{(0)}_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{kl}} V_{\mathbf{kl}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow}, \tag{1}$$

where $\epsilon^{(0)}_{\mathbf{k}}$ are the energies of the non-interacting single-particle states and $V_{\mathbf{kl}}$ are the matrix elements of the attractive effective interaction potential. The ground state will be denoted by $|BCS\rangle_0$. The Hamiltonian (1) is diagonalized by the Bogoliubov transformations, by writing $c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \equiv b_{\mathbf{k}} + (c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{k}})$, where $b_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}\rangle$, and assuming that $c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{k}}$ is small ($\langle\cdot\rangle$ is the average). Then $\mathcal{H}_{BCS}$ is expanded in terms of $c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{k}}$ and keeping only the first order we get

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \epsilon^{(0)}_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{kl}} V_{\mathbf{kl}} (c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} b_{\mathbf{l}} + b^*_{\mathbf{k}} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow} - b^*_{\mathbf{k}} b_{\mathbf{l}}). \tag{2}$$

We define the model Hamiltonian $\mathcal{H}_M = \mathcal{H} - \mu N$, which can be diagonalized to become [19]

$$\mathcal{H}_M = \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - \epsilon_{\mathbf{k}} + \Delta_{\mathbf{k}} b^*_{\mathbf{k}}) + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} (\gamma^\dagger_{\mathbf{k}0} \gamma_{\mathbf{k}0} + \gamma^\dagger_{\mathbf{k}1} \gamma_{\mathbf{k}1}), \tag{3}$$

where $\xi_{\mathbf{k}} \equiv \epsilon^{(0)}_{\mathbf{k}} - \mu$, $\epsilon_{\mathbf{k}} \equiv \sqrt{\xi^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}$ and $\Delta_{\mathbf{k}}$ is the energy gap,

$$\Delta_{\mathbf{k}} = -\sum_{l} V_{\mathbf{kl}} \langle c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow}\rangle. \tag{4}$$

The operators $\gamma^\dagger_{\mathbf{k}i}$ and $\gamma_{\mathbf{k}i}$ ($i = 0, 1$) are quasiparticle creation and annihilation operators, respectively (as defined in Ref. [19]), and are defined by the relations

$$c_{\mathbf{k}\uparrow} = u^*_{\mathbf{k}} \gamma_{\mathbf{k}0} + v_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}1}, \tag{5a}$$

$$c^\dagger_{\mathbf{k}\uparrow} = u_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}0} + v^*_{\mathbf{k}} \gamma_{\mathbf{k}1}, \tag{5b}$$

$$c^\dagger_{-\mathbf{k}\downarrow} = -v^*_{\mathbf{k}} \gamma_{\mathbf{k}0} + u_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}1}, \tag{5c}$$

$$c_{-\mathbf{k}\downarrow} = -v_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}0} + u^*_{\mathbf{k}} \gamma_{\mathbf{k}1}. \tag{5d}$$