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A generalized voter model with time-decaying memory on a multilayer network

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HIGHLIGHTS

- Multilayer network and time-decaying memory are incorporated into the voter model.
- Heterogeneity in peer pressure and time-decaying mechanism are detrimental to the consensus.
- Convergence time is governed by the coupled memory length and decay rate, and transitions are found.

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ABSTRACT

By incorporating a multilayer network and time-decaying memory into the original voter model, we investigate the coupled effects of spatial and temporal accumulation of peer pressure on the consensus. Heterogeneity in peer pressure and the time-decaying mechanism are both shown to be detrimental to the consensus. We find the transition points below which a consensus can always be reached and above which two opposed opinions are more likely to coexist. Our mean-field analysis indicates that the phase transitions in the present model are governed by the cumulative influence of peer pressure and the updating threshold. We find a functional relation between the consensus threshold and the decay rate of the influence of peer is found. As to the pressure. The time required to reach a consensus is governed by the coupling of the memory length and the decay rate. An intermediate decay rate may greatly reduce the time required to reach a consensus.

1. Introduction

In the physical world, the occurrence of phase transitions between a disordered system and an ordered system has been an important issue for many years, and it has been studied extensively [1–6]. For example, in the magnetic field, the spin-up

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and spin-down dynamics have been investigated by statistical physicists [7–13]. The spatial–temporal conditions for the magnetic order–disorder transition have been found. In terms of the coordination between two simultaneous processes is concerned, Li et al. investigated the synchronization of coupled phase oscillators [14]. The interfaces between different domains of synchronized oscillations have also been found. In addition, similar phenomena related to the order–disorder transition have been found in social and economic fields [15–19]. In a social context, competing for universal agreement is quite important in typical situations such as presidential elections and decision making in public affairs. The conditions required for reaching a consensus have been explored deeply by a variety of researchers, including sociologists, economists, statistical physicists, and mathematicians [20–23].

In the study of opinion dynamics, the binary-choice voter model and the Hegselmann–Krause model represent two distinctive types of opinion formation processes [24–29]. In the voter model, an individual's opinion has two possible states labeled as $+1$ and -1 . Depending on some type of updating rule, the individuals update their states sequentially or simultaneously. In the Hegselmann–Krause model, the individuals hold diverse opinions, which are expressed as real numbers within the range of $[-1, +1]$. According to a bounded confidence rule, the individuals gradually modify their opinions. Shao et al. studied the nonconsensus opinion model based on different networks [30]and they found that clusters of individuals with the same opinion could prevent themselves from being invaded by individuals with the opposite opinion. Fernandez-Gracia et al. investigated the driving forces that underlie the statistical features of US presidential elections [31], where they showed that social influence, mobility, and population heterogeneity have great impacts on the fluctuating features of the vote share.

Opinion dynamics provides us with insights into how a consensus can be reached within an interacting group. Previously, in the study of the dynamics of opinion formation, great efforts have been made to investigate the effects of comparatively stable interpersonal relationships and real-time interactions between different individuals. However, in the real world, an individual may have a diverse set of interpersonal relationships [32–36] by belonging to different types of social communities, such as relationships with relatives, colleagues, and business associates. The overlapping communities in complex networks were discussed by [14]. In addition, different communities usually have different impacts on personal opinion formation. The Matthew's effect demonstrates the inequality of advantages. Related studies of preferential attachment were reviewed by Perc [37]. In addition to the diversity of communities, another scenario related to opinion dynamics involves non-real time responses. A real-time interaction might not result in an immediate change but instead there may be a time-lagged change in personal behavior [38,39]. Thus, the coupled effects of diverse interpersonal relationships and the cumulative influence of individual interactions on opinion formation require in-depth study.

Multi-community interactions are usually modeled as a multi-layer network [40–43]. By incorporating a two-layer network into the SIR model, Zuzek et al. investigated the effects of isolation on the spread of an epidemic [44]. They found that the epidemic threshold increased with the length of the isolation period. Havlin et al. studied the effects of a percolation transition on interdependent networks [45] , where they showed that strong coupling between subnetworks leads to discontinuous phase transitions. Perc et al. studied the evolution of cooperation based on dynamic networks and multilayer networks [46–48] , where they found that the equilibrium and non-equilibrium processes are greatly affected by the interaction structures. Wang et al. studied interdependent network reciprocity in a public goods game [49] and found that cooperation can only be promoted when the coordination between different subnetworks is not disturbed. The coevolutionary dynamics of network interdependence and game strategy were investigated by Wang et al. [50] , who found that a coevolutionary system can reach the optimal conditions for cooperation. Recent studies related to multi-layered systems, including their structures, dynamics, evolution, and emergent properties, were reviewed by Kenett et al. [51].

The cumulative influence is usually modeled as memory. By incorporating the memory of past exposure to contagious influences into the original SIR model, Dodds et al. proposed a generalized epidemic model [52]and three classes of epidemic dynamics were found. Liu et al. studied the effects of memory on information propagation [53]. Compared with the situation where only internal or external influences exist, their coexistence makes the information spread more quickly and widely. To consider the time-lagged effect, a decaying confidence mechanism was introduced into the opinion formation model [54] and it was shown that local agreement was easier to achieve than a global consensus. Stark et al. investigated the effect of a time-dependent transition rate on opinion formation [55,56] and showed that an intermediate inertia rate may allow a consensus to be reached in much less time. The effects of memory on the evolution of social ties and spreading processes have been studied extensively [57–62].

In the present study, inspired by previous investigations of the multi-layer network model and the epidemic model with memory [41,52,55], we introduce a generalized voter model, which incorporates a multilayer network and memory of past influences. The spatial–temporal conditions required for a consensus are investigated extensively. Our main findings are as follows.

(1) Heterogeneity in peer pressure is detrimental to reaching a consensus. Under the condition that peer pressure is homogeneous, the effect of increasing multilayers is similar to the effect of increasing the random connections. It is easier to reach a . On consensus when each individual participates in more communities. Under the condition that peer pressure is heterogeneous, increasing multilayers is similar to reducing the mean influence of peer pressure. We find the critical values for multilayers below which increasing multilayers promotes the consensus and above which increasing multilayers leads to a nonconsensus state.
(2) The time-decaying mechanism is detrimental to the consensus and it increases the heterogeneous influence of peer pressure. Increasing the decay rate for the influence of peer pressure reduces the maximal value of the cumulative