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Pricing equity warrants with a promised lowest price in Merton's jump–diffusion model

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HIGHLIGHTS

- We present a pricing model for equity warrants in Merton's jump–diffusion environment.
- This proposed pricing model takes the presence of a promised lowest price into account.
- This paper introduces an estimation method for obtaining these desired variables based on observable data.
- This paper conducts an empirical study to ascertain the performance of our proposed model.

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ABSTRACT

Motivated by the empirical evidence of jumps in the dynamics of firm behavior, this paper considers the problem of pricing equity warrants in the presence of a promised lowest price when the price of the underlying asset follows the Merton's jump–diffusion process. Using the Martingale approach, we propose a valuation model of equity warrants based on the firm value, its volatility, and parameters of the jump component, which are not directly observable. To implement our pricing model empirically, this paper also provides a promising estimation method for obtaining these desired variables based on observable data, such as stock prices and the book value of total liability. We conduct an empirical study to ascertain the performance of our proposed model using the data of Changdian warrant collected from 25 May 2006 (the listing date) to 29 January 2007 (the expiration date). Furthermore, the comparison of traditional models (such as the Black–Scholes model, the Noreen–Wolfson model, the Lauterbach–Schultz model, and the Ukhov model) with our model is presented. From the empirical study, we can see that the mean absolute error of our pricing model is 16.75%. By contrast, the Black–Scholes model, the Noreen–Wolfson model, the Lauterbach–Schultz model, and the Ukhov model applied to the same warrant produce mean absolute errors of 92.24%, 45.38%, 87.34%, 76.12%, respectively. Thus both the dilution effect and the jump feature cannot be ignored in determining the valuation of equity warrants.

1. Introduction

As early as 1992, warrants were traded in China. Because of the illiquidity of the warrants market and the poor government supervision, warrant prices fluctuated widely and sometimes exceeded the prices of their underlying stocks. Therefore the Chinese government decided to close out all financial derivatives markets, including the short-lived warrants

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market. In 2005, the Chinese government launched the so-called Split Share Structure Reform. To expedite the share reform, the China Securities Regulatory Commission considered to compensate investors in the floating shares for their potential losses by permitting corresponding firms to issue warrants. As a consequence, warrants have become popular in China since 2005. Moreover, China's warrant market had become the largest warrant market in the world in terms of trading volume since 2006. Although the Chinese warrant market is very large, only a few studies on the price behavior of warrants are known (see, for instance, Refs. [1–3]).

The goal of this paper is to contribute to extant literature by pricing one special type of warrants, namely equity warrant with a promised lowest price, in the Merton's jump–diffusion process. Actually, according to the type of issuers, two different types of warrants are actively traded in China: covered warrants and equity warrants. Covered warrants, which are issued by a third party (e.g., an investment bank), give the holder the right, but not the obligation, to buy or to sell an underlying asset at a pre-determined price by a predetermined date. Moreover, on the expiration date, the issuer of covered warrants just sells stocks to warrants holder if it is stock settlement, or the issuer of covered warrants just pay the difference between strike price and the market value of the underlying asset in cash to the warrants holder if it is cash settlement. Thus, the number of outstanding shares will not be changed when covered warrants are exercised. Hence, the pricing models of European options can be used to price European covered warrants since European covered warrants have no dilution effect when they are exercised.

However, equity warrants, issued by the underlying firm itself, entitle holders to purchase equity securities from the underlying firm at a predetermined price. Unlike covered warrants, the exercise of the equity warrant requires the issuance of new stocks by the corresponding firm, resulting in dilution. Hence, pricing equity warrants is more complicated than pricing covered warrants. To allow for possible dilution when pricing equity warrants, in the literature, several theoretical and empirical models for equity warrants have been proposed since 1970s. Indeed, Ref. [4] stated that the Black–Scholes option pricing model (see Ref. [5]) was initially intended to value warrants. Ref. [6] presented a standard pricing model for equity warrants which corrected Black–Scholes model for dilution. Ref. [7] provided a new pricing model for equity warrants, which took the dilution into account for valuing equity warrants. Later, Refs. [8,9] held that the correction for dilution was required. These afore-mentioned research primarily treated an equity warrant as a contingent claim on the total "value of the firm" in deriving a valuation formula. However, the total value of the firm is not generally observable, which makes it difficult to apply the corresponding valuation models. To overcome this difficulty, Ref. [10] proposed a warrant-pricing procedure based on the stock price and the stock return variance. After that, Ref. [11] developed an algorithm that generalized the model of Ref. [10] for the case of the warrant ratio being distinct from unity. In fact, the pricing approach, suggested by Ref. [11], involves the numerical solution of an iterative algorithm based on the stock price and the stock return volatility. Since the publication of this substantial paper, the Ukhov model of Ref. [11] has subsequently become the standard methodology for pricing equity warrants. Many improvements and developments on the Ukhov model of Ref. [11] have been considered (see, Refs. [12–15]). Moreover, a large number of empirical studies have shown that financial time series usually exhibit properties of self-similarity, heavy tails, long-range dependence in auto-correlations and cross-correlations, volatility clustering(see, Refs. [16–32]). On the other hand, empirical studies have also been conducted in the literature. For example, Ref. [7] tested various valuation models of equity warrants based on warrants listed on the New York and American Stock Exchanges from 1971 to 1980. The comparison of the alternative pricing methods with a sample of US warrants was considered by Ref. [33]. A similar empirical study of 16 German stocks over the period 1979–1990 was also conducted by Ref. [10]. Ref. [34] used the dilution-adjusted Black–Scholes model in empirical study of warrants. Ref. [35] examined the price performance of call warrants in China's securities market. They found that trading mechanism constraints in China's securities market prevent rational investors from driving the prices of some call warrants to a reasonable level.

These research mentioned above considered the problem of pricing equity warrants under the assumption that the prices of the underlying assets follow some diffusion-type processes, in particular the geometric Brownian motion. However, documentation from various empirical studies has shown that these diffusion-type models are inadequate, both in relation to their descriptive power, as well as for the mispricing that they might induce. The key reason for this mispricing is that these diffusion-type processes cannot capture the dynamics of firm behavior. In fact, a pure diffusion model fails to reflect many empirical phenomena, such as the leptokurtic feature, volatility clustering effect, and implied volatility smile. Since all of these features play key roles in modeling the process for the firm asset value, one will produce some mistakes because of ignoring them. Consequently, in the past decades, there have been elaborate efforts by researchers to build models that have taken these empirical stylized facts into consideration. One model to consider these empirical stylized facts is by introducing some extra randomness. For instance, the authors can add a jump process to the diffusion model. Specifically, Ref. [36] argued that the fluctuations of asset prices can be decomposed as the sum of "normal" vibrations, caused by temporary imbalance of supply and demand and other new information that causes marginal changes in the stock value, and "abnormal" vibrations, caused by the arrival of important new information which generates occasional and large impact on price. Furthermore, Ref. [36] modeled the normal and abnormal variations by a standard geometric Brownian motion and a Poisson process, respectively, and derived an option pricing formula. Subsequent work has been moving towards considering other or more general Lévy process, such as the double exponential jump–diffusion model (see, Ref. [37]) and the mixed-exponential jump–diffusion model (see, Ref. [38]). As a consequence, many scholars have considered the problem of pricing options in jump–diffusion models (see, for example, Refs. [39–42] and references therein). Similarly, to catch these random jumps in the firm value, some scholars developed some models that incorporated the occurrence of rare jumps in underlying asset. This was done by Ref. [43] which used the jump–diffusion model that was developed by Merton in Ref. [44]. Ref. [45] extend