Physica A 458 (2016) 1–8

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Average receiving scaling of the weighted polygon Koch networks with the weight-dependent walk

Dandan Ye, Meifeng Dai *, Yanqiu Sun, Shuxiang Shao, Qi Xie

Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, PR China

HIGHLIGHTS

- Weighted polygon Koch networks with a weight factor are presented.
- Multi-layered division method is used to divide the weighted polygon Koch networks.
- Average receiving time is affected by the weight factor.

ARTICLE INFO

Article history:
Received 15 January 2016
Available online 12 April 2016

Keywords:
Self-similarity
Average receiving time
Weighted polygon Koch network
Weight factor
Weight-dependent walk
Recursive division

ABSTRACT

Based on the weighted Koch networks and the self-similarity of fractals, we present a family of weighted polygon Koch networks with a weight factor r $(0 < r \leq 1)$. We study the average receiving time (ART) on weight-dependent walk (i.e., the walker moves to any of its neighbors with probability proportional to the weight of edge linking them), whose key step is to calculate the sum of mean first-passage times (MFPTs) for all nodes absorpt at a hub node. We use a recursive division method to divide the weighted polygon Koch networks in order to calculate the ART scaling more conveniently. We show that the ART scaling exhibits a sublinear or linear dependence on network order. Thus, the weighted polygon Koch networks are more efficient than expended Koch networks in receiving information. Finally, compared with other previous studies' results (i.e., Koch networks, weighted Koch networks), we find out that our models are more general.

1. Introduction

In the past decade, since many systems in the real world can be described and characterized by complex networks [1–3], complex networks have become a powerful and common tool. Besides, fractals are an important concept characterizing the features of real systems [4], because they can model a broad range of objects in nature and society [5].

It has attracted a surge of interest from the scientific community that fractal structures are converted into complex networks [6]. Xi et al. obtain the asymptotic formula for average path length of the Sierpinski gasket constructed by a new method [7]. They also introduce multiple hubs based on scale-free and small-world networks and present the trapping problem on them [8,9]. Zhang et al. proposed Koch networks based on Koch snowflake [10–13]. They presented some key properties, such as degree correlations, a high clustering coefficient, small average path length and power-law distribution. And they investigated random walks with an immobile trap fixed at a hub node with the highest degree performing on Koch networks [14,15]. For the pseudofractal scale-free web, they also study analytically the related first passage properties [16].

* Corresponding author.
E-mail address: daimf@mail.ujs.edu.cn (M. Dai).

http://dx.doi.org/10.1016/j.physa.2016.03.097

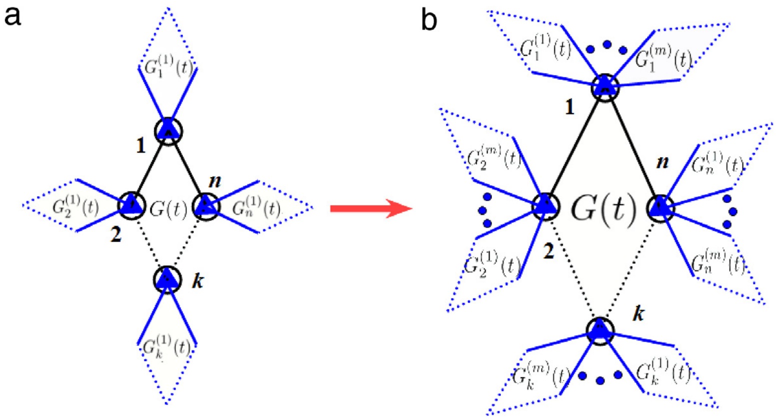

Fig. 1. Construction method of the weighted polygon Koch network. The open circle and triangles represent Node i ($i = 1, 2, \ldots, n$) of $G(t)$ and $a^{i,1}, \ldots, a^{i,m}$ ($i = 1, 2, \ldots, n$) of $G_j^i(t)$ ($j = 1, 2, \ldots, m$), respectively. (a) Case $m = 1$. (b) Case $m > 1$.

Enlightened by Koch networks, there are many expanded works [17–22]. For example, Zhang et al. [17–19] studied the generalized Koch networks and transformed the initial state of the Koch network from a triangle to an n-polygon. They investigated the random walks and the trapping on the generalized Koch networks and obtained the exact solution of the MFPT, which shows that the MFPT grows linearly with the increasing order of the networks, and the MFPT increases with network parameter n [19]. These above papers are based on unweighted networks, i.e., edges among nodes are either present or absent, represented as binary states. In fact, as we know, many networks are intrinsically weighted. Weighted networks represent the natural framework to describe natural, social, and technological systems, in which the intensity of a relation or the traffic between elements is an important parameter [23]. In general terms, weighted networks are extension of networks or graphs [1,24,25], in which each edge between nodes i and j is associated with a variable w_{ij}, called the weight. Taking the airport networks [26] for example, the number of passengers can directly image the status of airlines. Similarly, in Internet networks [27], the load of information traffic along edges or through routers can reflect the importance of edges or routers in traffic transportation. The heterogeneity of weights affects dynamical processes taking place on a network, which include random walk [28], and so on. It is, thus, of theoretical and practical interests to construct weighted networks and investigate random walks on them.

Diffusion is a key element of a large set of phenomena occurring on natural and social systems modeled in terms of weighted complex networks. Assuming that the diffusion process is local, there are three most general kinds of random walks: random walk, weigh-dependent walk and strength-dependent walk. A random walker may choose one of its neighboring edges at the same probability (random walk). In weighted networks, however, the walker will choose an edge according to its weight or the strength of the node connected by it, i.e. weight-dependent walk or strength-dependent walk. The average receiving time (ART) is the sum of mean first-passage times (MFPTs) for all nodes absorpt at the trap located at a hub node. In 2012, Dai et al. [29,30] proposed a family of weighted Koch networks and discussed the average receiving time and the average weighted shortest path.

Motivated by the generalized Koch networks [17,18] and the weighted Koch networks [29,30], we firstly construct the weighted polygon Koch networks. The mean first-passing time (MFPT) is the expected first arriving time for the walks starting from a source node to a given target node. In the weighted n-polygon Koch networks, n hub nodes are successively labeled as Node i ($i = 1, 2, \ldots, n$). We arbitrarily choose one of n hub nodes as the trap, i.e., Node 1. Let $F_i(t)$ be the MFPTs from Node i ($i = 2, 3, \ldots, n$) to the trap. In the weighted triangle Koch networks ($n = 3$), the MFPTs from other two hub nodes to the trap are the same because of their equal status. However, when $n \geq 4$ in the weighted n-polygon Koch networks, $F_i(t)$ changes with relative location of Node i ($i = 2, 3, \ldots, n$) against to the trap. Then, we study the average receiving time (ART) on weight-dependent walk in order to discuss the efficiency of a hub node receiving information on them. We use recursive division method to divide the weighted polygon Koch networks in order to calculate the average receiving time (ART) on weight-dependent walk conveniently. The obtained result shows that the efficiency of hub node receiving information is affected by the weight factor.

2. The weighted polygon Koch networks

This section mainly describes the construction of the weighted polygon Koch networks. Based on the generalized Koch networks [17–19] and the weighted Koch networks [29,30], we built the weighted polygon Koch networks in an iterative way. A positive real number $0 < r \leq 1$ is needed.

Let $G(t)$ be the weighted polygon Koch networks of the tth generation. Then, the weighted polygon Koch networks can be created in the following way (see Fig. 1): Initially ($t = 0$), $G(0)$ consists of n ($n \geq 3$) nodes and n edges with unit weight forming an n-polygon, where the initial n node in $G(0)$ are labeled as Node i ($i = 1, 2, \ldots, n$). For $t \geq 1$, $G(t + 1)$ may be