Physica A 458 (2016) 67–75

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Geometric measure of quantum correlation: The influence of the asymmetry environments

Qinsheng Zhu [a], Changchun Ding [a,*], Shaoyi Wu [a], Wei Lai [b]

[a] *School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu 610054, PR China*
[b] *Experimental Middle School of Chengdu Economic and Technology Development Zone, Chengdu 610054, PR China*

HIGHLIGHTS

- The geometric quantum discord (GQD) of a two qubits system is investigated.
- The behaviors of GQD under the asymmetry environments are displayed and discussed.
- The revival phenomenon is depressed with increasing temperature T.
- The coupling parameters q_1 and q_2 can improve revival phenomenon.
- The GQD shows the reverse (order) phenomena at lower (higher) temperatures.

ARTICLE INFO

Article history:
Received 9 October 2015
Received in revised form 14 February 2016
Available online 13 April 2016

Keywords:
Quantum discord
Entanglement
Non-Markovian process

ABSTRACT

The quantum correlation in open quantum systems is of fundamental and practical importance for quantum information processing and controllable nanometer devices. And the properties of quantum correlation can be influenced by the information flow between systems and environments. In this study, we investigated the geometric measure discord of quantum correlation of a two qubits system, interacting with two independent and intrinsic interacting spin-environments, respectively. Based on the asymmetry environments with comparable parameters, the different properties of the geometric measure of entanglement and quantum discord are displayed and discussed for initial Bell states.

1. Introduction

As a basic theory, the superposition principle and the tensorial structure of the Hilbert space have been widely applied to describe a composite quantum system in quantum mechanics. Particularly, some quantum properties which can hardly be generated by a classical system can be exhibited by using the above two principles for a composite quantum systems, e.g. entanglement and quantum correlation. The earliest quantum correlation research can date back to the study of entanglement [1], which is still significant in quantum information field and has been utilized as a crucial resource for communication and computation [2]. It had been regarded as true that entanglement is equivalent to quantum correlation for many years until the concept of “quantum discord” was introduced by Ollivier and Zurek [3] and Henderson and Vedral [4]. In fact, entanglement does not account for all nonclassical correlations and that even separable states usually contain correlations that are not entirely classical [3,5]. In other words, entanglement only represents a portion of the quantum correlations in realistic quantum systems, and now there is universal consensus that entanglement entirely

---

[*] Corresponding author.
E-mail address: ccding626@163.com (C. Ding).

http://dx.doi.org/10.1016/j.physa.2016.04.004

captured quantum correlation only for a global pure state [6]. Therefore, finding valuable and general traits of these quantum correlation resources constitutes a clear potential and useful methodology of their practical exploitation, particularly in dynamical evolution aroused by the unavoidable interactions between quantum systems and the surrounding environments (local measurement or operator of quantum systems) [7–9].

For different interactions between environments and systems, the dynamical evolution may generally show Markovian and non-Markovian behaviors. Under Markovian environments, quantum correlation exhibits monotone decrease or disappearance at a finite time [10–13]. However, some special natures, e.g. revivals of quantum correlations, have been aroused by the non-Markovian dynamics process. For classical environments, revival phenomena can occur in absence of both system–environment back-action and storing of quantum correlations due to information flows back and forth from the system to the environment [14–18]. But for quantum environments, there are the information flux between the system and environments and system–environment back-action [18–39]. In order to understand the principle of the changing behaviors of correlation, e.g. quantum correlation, classical correlation and total correlation, two equivalent measurements and quantization methods (i.e., geometric distance approach [6] and the relative entropy method [3,4]) have been used to research these correlations and related physical problems (e.g. quantum phase transition [40]). Recently, some theoretical and experimental results about correlation have been obtained by using the above two methods. For example, the properties of quantum discord, entanglement and corresponding classical and total correlation were discussed based on the von Neumann entropy using some theoretical and experimental models [5,41–45,6,46,48,47]. In geometric distance approach, several valid geometric distances of quantum correlation in mathematics were proposed, e.g. the Bures distance [48], the Hellinger distance [49] and Schatten one-norm discord (the trace distance) [50]. Although there maybe some inherent problem [51], it showed the relationship between the correlations and entanglement from space perspective, e.g., a novel definition of geometric distance measure was given to quantify entanglement and quantum correlations [48,6], and the freezing [52] and revivals phenomena of correlations were also shown by geometric measures distance for some open quantum systems. In this paper, we mainly discussed the affect of the parameters of the asymmetric environment for quantum correlations in order to be helpful for the controllability of the quantum correlations.

Because of the inevitable interactions between the environment and systems for a realistic physical system, it is important to research the influence of environmental parameters on quantum correlations and entanglement in open quantum systems. According to previous research model for quantum correlations, these models can be largely classified into two categories, i.e., one for the models consisting of a common environment [53] and another for the models of two qubits system interacting with two independent environments [47]. Sometimes, one of the two environments may not both exist like photonic open system model researched by Bi-Heng Liu [20]. Usually, merely the zero temperature and intrinsic non-interacting environments were considered in these researches [53]. In this work, based on the interest in asymmetric and intrinsic interacting environments [32], quantum correlation and entanglement of two non-interacting spin qubits systems which interact respectively with two independent and intrinsic interacting spin-environments are investigated by the geometric measure method. Meanwhile, the influences of asymmetric environmental parameters (temperature T and intrinsic interacting strengthen q) on quantum correlation and entanglement are discussed under initial Bell-diagonal states conditions.

This paper is organized as follows. In Section 2, we briefly review the concept of Geometric measure of Quantum Discord (GQD) and entanglement (GE). In Section 3, the model Hamiltonian is given and the exact solution of reduce density matrix of the system is obtained under non-Markovian condition. In Section 4, for initial Bell-diagonal states, the properties of GQD and GE are researched under the asymmetry environments parameters (temperature T and intrinsic interacting strengthen q).

2. A review of geometric measure of quantum discord and entanglement

For quantum correlations, there are some measurement and quantification methods based on entropic quantities, including the one-way quantum deficit [54] and the quantum discord [3,4]. The quantum discord describes a quantum correlation of a bipartite state ρ with marginals ρ_a and ρ_b on a system $H_a \otimes H_b$ and can be expressed as:

$$\begin{aligned} Q(\rho) &= \min_{\prod^a} \left\{ I(\rho) - I\left(\prod^a(\rho) \right) \right\} \\ \prod^a(\rho) &= \sum_k \left(\prod_k^a \otimes I^b \right) \rho \left(\prod_k^a \otimes I^b \right). \end{aligned} \tag{1}$$

Here the minimum is in terms of von Neumann measurements ($\prod^a = \{\prod_k^a\}$) on subsystem a. $I(\rho)$ is the quantum mutual information ($I(\rho) = S(\rho^a) + S(\rho^b) - S(\rho)$), and $S(\rho)$ is the von Neumann entropy ($S(\rho) = -\mathrm{tr}\rho \ln \rho$) and I^b is the identity operator on H^b.

Another measurement method of quantum correlation is geometric approach. By choosing a metric over the space of quantum states, the distance to the nearest zero-discord state can be found. According to Dakic's proposal [55], the Hilbert–Schmidt measure of discord can be defined as:

$$D(\rho) = \min_{\chi} \left(\|\rho - \chi\|_1^2 \right) \tag{2}$$