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Quantum games on evolving random networks

Łukasz Pawela

Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, Bałtycka 5, 44-100 Gliwice, Poland

HIGHLIGHTS

- Quantum social dilemmas on evolving random networks are studied.
- The influence of the game parameters on strategy distribution is analyzed.
- Results show that quantum strategies dominate the network in most cases.

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ABSTRACT

We study the advantages of quantum strategies in evolutionary social dilemmas on evolving random networks. We focus our study on the two-player games: prisoner's dilemma, snowdrift and stag-hunt games. The obtained result show the benefits of quantum strategies for the prisoner's dilemma game. For the other two games, we obtain regions of parameters where the quantum strategies dominate, as well as regions where the classical strategies coexist.

1. Introduction

Game theory is a widely studied branch of science with broad applications in a plethora of fields. These range from biology to social sciences and economics. It has been especially useful in the study of social dilemmas, *i.e.*situations where the benefit of the many should be put in front of the benefit of the individual. One of the most frequently studied approaches in this context is the evolutionary game theory [1]. The field of evolutionary games has since evolved and now studies not only games on regular grids, but also on complex graphs [2]. Recently, there are studies focused on studying social dilemmas on evolving random networks [3,4].

In quantum game theory [5–8], we allow the agents to use quantum strategies alongside classical ones. As this is a far larger set of possible players' moves, it offers the possibility of much more diverse behavior. The most outstanding example of this is the fact that if only one player is aware of the quantum nature of the game, he/she will never lose in some types of games. This is found, for example, in social dilemmas, where quantum strategies introduce a miracle move [9]. This is a strategy that always wins against any classical strategy.

If both players are aware of the nature of the game, one of them might still cheat by appending additional qubits to the system [10]. When we take decoherence into account, the game behavior changes. In particular, the well known Nash equilibrium of a game can shift to a different strategy [11]. On top of this, there also exists a quantum version of the Parrondo's paradox [12,13]. Finally, there exist the quantum pseudo-telepathy games. In these games, players utilizing quantum strategies and quantum entanglement may seem to an outside observer as they are communicating telepathically [14–16].

E-mail address: lpawela@iitis.pl.

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The combination of the fields of quantum game theory and evolutionary games has led to numerous results [17–20]. There exist cases where the quantum strategies dominate the entire network. In this work, we aim to study the behavior of three quantum games on evolving random networks: prisoner's dilemma, snowdrift and stag-hunt games. The transition between these games will be achieved by manipulating the parameters of the game. Games on evolving networks have been studied in the classical [2,4] as well as quantum settings [21]. The evolution of the network can be seen as aging of the agents.

This paper is organized as follows. In Section 2, we introduce quantum games. In Section 3, we discuss the model of networks used in this work. Section 4 contains the results along with discussions. Finally, in Section 5, we draw the final conclusions.

2. Quantum games

We call a game a *quantum game* if the players participating are allowed to use quantum strategies. By quantum strategies we understand moves that have no analogue in classical game theory, but have a good interpretation in the realm of quantum mechanics. We will focus on two-player games. Henceforth, we will call the players Alice and Bob.

2.1. General concepts

Formally, a two-player quantum game is a tuple $\Gamma = (\mathcal{H}, \rho, S_A, S_B, P_A, P_B)$. Here, $\mathcal{H}$ is a Hilbert space of the system used in the quantum game, and ρ is the system's initial state. Note that ρ is a positive operator with unit trace, *i.e.* $\rho \geq 0$ and $\mathrm{Tr}\rho = 1$. Allowed Alice's and Bob's strategies are given by the sets S_A and S_B, respectively. Their payoff functions are given by P_A and P_B. They are functions mapping players' strategies to numerical values. In general, the strategies $s_A \in S_A$ and $s_B \in S_B$ can be any quantum operations. A definition of a quantum game may contain additional rules like the ordering of players or the number of times they are allowed to make a move.

By analogy to the classical game theory, we may define the following quantities in quantum game theory. We will call a strategy s_A the *dominant strategy* of Alice if $P(s_A, s'_B) \geq P(s'_A, s'_B)$ for all $s_A \in S_A, s'_B \in S_B$. Following this pattern, we may define a dominant strategy for Bob. A pair of strategies (s_A, s_B) is an equilibrium in dominant strategies if and only if s_A and s_B are Alice's and Bob's dominant strategies. A pair of strategies is *Pareto optimal* if it not possible to increase one player's payoff without decreasing the other player's payoff. Finally, we define a Nash equilibrium as a set of strategies, such that no player can do better by unilaterally changing their strategy. This will be further discussed when we introduce the quantum prisoner's dilemma game.

2.2. Quantizing the prisoner's dilemma, stag hunt and snowdrift games

The setup in the quantum case is as follows. Each player is given by a referee a single qubit and may only operate on it locally. Hence, we have $s_A, s_B \in SU(2)$, where $SU(2)$ is the set of unitary 2×2 matrices with unit determinant. Initially, the qubits are entangled:

$$|\phi\rangle = J|00\rangle, \tag{1}$$

where J is the entangling operator [22]:

$$J = \frac{1}{\sqrt{2}} \left(\mathbb{1} \otimes \mathbb{1} + \mathrm{i}\sigma_x \otimes \sigma_x \right). \tag{2}$$

Here σ_x is the Pauli matrix:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3}$$

Next, the players apply their respective strategies U_A and U_B and the untangling operator $J^\dagger$ is applied by the referee. Here, $J^\dagger$ denotes the Hermitian conjugate of J. The final state of the system is

$$|\psi\rangle = J^\dagger (U_A \otimes U_B) J|\phi\rangle. \tag{4}$$

This is shown as a quantum circuit in Fig. 1.

The payoff matrix of a two player game with cooperators C and defectors D is shown in Table 1. In the table, R is the reward, P is the punishment for mutual defection, S is known as the sucker's payoff and finally the parameter T is the defector temptation. In our analysis, we set $R = 1$ and $P = 0$. The remaining two parameters range is $S \in [-1, 1]$ and $T \in [0, 2]$. When $T > R > P > S$ we get a social dilemma—the prisoner's dilemma. Note that on the one hand, in this case the strategy profile (C, C) is Pareto optimal, but on the other hand the profile, (D, D) is a Nash equilibrium. Hence, we have the dilemma. Next, when $T > R > S > P$ we get the snowdrift game. Finally, when $R > T > P > S$ we get the stag-hunt game.