Physica A 430 (2015) 136–147

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

State impulsive control strategies for a two-languages competitive model with bilingualism and interlinguistic similarity ☆

Lin-Fei Nie [a,*], Zhi-Dong Teng [a], Juan J. Nieto [b,c], Il Hyo Jung [d]

[a] College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, PR China

[b] Departamento de Análisis Matemático and Instituto de Matemáticas, Universidade de Santiago de Compostela, 15782, Santiago de Compostela, Spain

[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[d] Department of Mathematics, Pusan National University, Busan 609-735, Republic of Korea

HIGHLIGHTS

- A novel two languages competitive model with state pulse control strategies is introduced.
- The model includes two control threshold values, it is different from the previous models.
- Sufficient conditions on the existence and stability of stable periodic solutions are presented.
- How the state-dependent pulse control strategies affect the endangered language is discussed.
- Numerical simulations are carried out to illustrate the theoretical results.

ARTICLE INFO

Article history:
Received 4 December 2013
Received in revised form 9 February 2015
Available online 3 March 2015

Keywords:
Language competitive model
State-dependent impulsive control
Order-*k* periodic solution
Multiple periodic solutions
Orbital stability

ABSTRACT

For reasons of preserving endangered languages, we propose, in this paper, a novel two-languages competitive model with bilingualism and interlinguistic similarity, where state-dependent impulsive control strategies are introduced. The novel control model includes two control threshold values, which are different from the previous state-dependent impulsive differential equations. By using qualitative analysis method, we obtain that the control model exhibits two stable positive order-1 periodic solutions under some general conditions. Moreover, numerical simulations clearly illustrate the main theoretical results and feasibility of state-dependent impulsive control strategies. Meanwhile numerical simulations also show that state-dependent impulsive control strategy can be applied to other general two-languages competitive model and obtain the desired result. The results indicate that the fractions of two competitive languages can be kept within a reasonable level under almost any circumstances. Theoretical basis for finding a new control measure to protect the endangered language is offered.

☆ This research has been partly supported by the National Natural Science Foundation of China (Grant No. 11461067, 11402223 and 11271312), the Ministerio de Economia y Competitividad (Spain), project MTM2010-15314, and co-financed by the European Community fund FEDER.

* Corresponding author. Tel.: +86 991 858 5505; fax: +86 991 858 5505.
E-mail addresses: lfnie@163.com (L.-F. Nie), zhidong@xju.edu.cn (Z.-D. Teng), juanjose.nieto.roig@usc.es (J.J. Nieto), ilhjung@pusan.ac.kr (I.H. Jung).

http://dx.doi.org/10.1016/j.physa.2015.02.064

1. Introduction

Language diversity, one of the best examples of human intelligence, has been the object of scientific work from many points of view, including physics and mathematics. Particularly, physicists modeled the language learning, language change, and application as well as the structure of natural languages (see, e.g., Refs. [1–8] and the references therein). Among them, the initial and simplest mathematical model of two languages competitions described by ordinary differential equations, which was proposed by Abrams and Strogatz [9]. In it, they assumed that there is no spatial or social structure in the populations, in which all speakers are monolingual and the attractiveness of a language increase in both its number of speakers and its perceived status. The theoretical results successfully fitted to historical data on the competition between Scottish Gaelic and English, Welsh and English, and Quechua and Spanish, among others language pairings.

The truth, however, is that multilingual speakers outnumber monolingual speakers in some areas of the world. Multilingualism is becoming a social phenomenon governed by the needs of globalization and cultural openness [10,11]. Owing to the ease of access to information facilitated by the Internet, individuals' exposure to multiple languages is becoming increasingly frequent thereby promoting a need to acquire additional languages [12]. In Spain, for example, where Castilian Spanish is the official language throughout the state, but in certain regions is co-official with another language (mainly Galician, Basque, or Catalan). Individual bilingualism is common in communities with more than one official language. So for this reason, Mira et al. [5,13] studied a generalization of the Abrams–Strogatz model that allows for bilingual as well as monolingual speakers of the competitive languages, and that includes a parameter that represents the ease of bilingualism.

Throughout most of the history of languages, many languages appear, evolve and disappear. With partial regional results tallied, the numbers of languages in the world vary between 6000 and 7000. However, with the progress of the globalization, thousands of the world languages are vanishing at an alarming rate, with 90% of them being expected to disappear with the current generation as a result of language competition [14–19]. How to cope with the forceful language and preserve the diversity and rich resources of the national language are a key issue worthy of exploration. In this regard, exploring an effective and easily implemented control measure to preserve some of the endangered languages is significant both theoretically and practically.

In this paper, we intend to make a contribution by introducing an extended two-languages competitive model to assess the impact of state-dependent impulsive control strategy. We focus on a control model of two-languages competition that allows bilingualism introduced in Ref. [5] and whose most interesting results also were derived in Refs. [13,20–22]. By analyzing the model, we investigate how state-dependent impulsive control strategy protects some of the endangered languages.

The remaining parts of this paper are structured as follows. In the next section, we introduce a novel two-languages competitive model with state-dependent impulsive control strategies. Section 3 provides the existence and orbital stability of positive periodic solutions for this model. Numerical simulations are carried in Section 4 for illustration. Some concluding remarks are described in Section 5.

2. The model and preliminaries

A classical autonomous two-languages competitive model was developed by Mira et al. [5,13]. In this model, the total population is divided into language X speakers, language Y speakers and bilingual speakers B, whose dynamics are modeled by the standard system of differential equations

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = yP_{YX} + bP_{BX} - x(P_{XY} + P_{XB}) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = xP_{XY} + bP_{BY} - y(P_{YX} + P_{YB}) \\ \dfrac{\mathrm{d}b}{\mathrm{d}t} = xP_{XB} + yP_{YB} - b(P_{BY} + P_{BX}) \end{cases} \tag{1}$$

with

$$\begin{aligned} &P_{XB} = ck(1-s)(1-x)^a, \qquad P_{BX} = P_{YX} = c(1-k)s(1-y)^a, \\ &P_{YB} = cks(1-y)^a, \qquad P_{BY} = P_{XY} = c(1-k)(1-s)(1-x)^a \end{aligned} \tag{2}$$

where, letters x, y refer to the fractions of two languages speakers in population, respectively, b denotes the fraction of bilingual speakers (with $x + y + b = 1$), c is a normalization factor related to the time scale, a is the power parameter, s denotes the relative status of language X and by $1 - s$ that of language Y, and k is the probability that the disappearance of a monolingual speaker of language X (respectively Y) will be compensated for by the appearance of a bilingual rather than by a monolingual speaker of language Y (respectively X). The model determined by Eq. (1) with the probabilities given by Eq. (2) has been studied extensively, and some of them can be found in Refs. [23,5,13,21,7] and the references therein.

Generally speaking, for two competitive languages, when the fraction of one language in the population is higher, then the other language will face the risk of gradually vanishing. That is, one is the dominant language and the other as the endangered language. In this case, the relevant government departments or other organizations could take deliberate reasonable