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Transition probability, dynamic regimes, and the critical point of financial crisis

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HIGHLIGHTS

- Birth–death process describes the market instability induced by social interaction.
- Time-varying transition probability shows calm and turbulent regimes of market.
- There is a visible link between liberalization policy and market nonlinearity.
- The critical point in 2008 matches historical records of the Subprime Crisis.

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ABSTRACT

An empirical and theoretical analysis of financial crises is conducted based on statistical mechanics in non-equilibrium physics. The transition probability provides a new tool for diagnosing a changing market. Both calm and turbulent markets can be described by the birth–death process for price movements driven by identical agents. The transition probability in a time window can be estimated from stock market indexes. Positive and negative feedback trading behaviors can be revealed by the upper and lower curves in transition probability. Three dynamic regimes are discovered from two time periods including linear, quasi-linear, and nonlinear patterns. There is a clear link between liberalization policy and market nonlinearity. Numerical estimation of a market turning point is close to the historical event of the US 2008 financial crisis.

1. Introduction

How to diagnose the nature of business cycles and financial crises is still an open issue in economics and finance [1,2]. There is serious debate on the nature of financial crises. The equilibrium school believes in a self-stabilizing market and attributes external shocks as the only source of cycles and crises [3–7], while the disequilibrium school mainly considers various mechanisms of market instability [8–10]. Current econometric analysis has difficulty in diagnosing crises since its analytical foundation is static probability distribution. The equilibrium school assumes the Gaussian distribution or i.i.d. with a finite mean and variance [4], while the disequilibrium school introduces non-Gaussian distributions, such as the Levy distribution, fat tails and power law [7,11–13]. The disequilibrium approach is attractive in empirical analysis but impotent in policy studies, since it has little means to prevent external shocks. The third approach is a computational simulation of heterogeneous agents [14]. This approach can generate unstable patterns suggested by behavioral economics, but is hard to apply in empirical analysis of a financial crisis.

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Our research strategy is to choose a proper degree of abstraction, so that it is simple enough to explain key empirical observations but general enough to integrate existing theories. We have solid evidence that financial movements are nonlinear and non-stationary in nature [2]. Therefore, we develop a general approach in analyzing a non-stationary time series, so that the equilibrium, stationary, and linear scenario are special cases of non-equilibrium, non-stationary, and nonlinear situations. Historical features can be used to test crisis theory [15]. Our breakthrough in diagnosing financial crisis is achieved by replacing the static model of the representative agent by the time-varying non-Gaussian probability distribution of population dynamics. The master equation approach is widely used in statistical mechanics in physics and chemistry, which is called the social dynamics in studies of interacting agents in sociology and economics [16–19]. According to this approach, equilibrium analysis mainly considers the first (mean value) and second moment (variance), while non-equilibrium situations study social behavior with higher moments [20].

In a recent paper, we show that high moments representation can provide effective warning signals of a regime-switch or an upcoming financial crisis [21]. We will further demonstrate that the changing patterns in transition probability can diagnose the nature and cause of the 2008 financial crisis. Methodologically speaking, the time-varying probability distribution is richer than the static feature from the representative agents, but simpler than the computational model of heterogeneous agents in empirical and theoretical analysis. Time-varying probability distribution can be described by non-linear transition probability in a master equation for the birth–death process. The transition probability in different historical time windows provides valuable information on structural stability of dynamical markets, while the high moments representation may provide timely warning of coming crisis.

The technical issue is how to derive the transition probability from empirical observation and economic mechanisms since the Gaussian type distribution in equilibrium physics may not be valid for social systems [16,22]. For example, the herd behavior in a social population may generate a bi-modular distribution [23–25]. Here, we do not make any ad hoc behavioral assumptions for market dynamics. Instead, we take a phenomenological approach to derive the transition probability from empirical data. We develop a numerical algorithm to establish a link between the master equation and the empirical estimation of transition probability. Herd behavior and bi-modular distribution can be explained by logistic interaction in transition probability [2,24].

We estimate the transition probability in two separate periods: one period, 1950–1980, was dominated by Keynesian policy and New Deal regulation; and the period of 1981–2010 is the liberalization era started by the Reagan-administration and includes the 2008 crisis. In each period, we assume that the transition probability is the function of market states (i.e. current prices), but independent of time. This procedure is similar to the two-stage econometric analysis. The difference is attributed to different mathematical representations. Econometric analysis is based on a matrix, while a probability distribution needs to solve partial differential equations. A more advanced mathematical representation may reveal more patterns in complex dynamics.

Through empirical analysis, we introduced a quantitative indicator of population behavior: the transition probability between neighboring states of price indexes. We discovered the nonlinear shape of transition probability for the period of liberalization and crisis, which is rooted in trading behavior. We found a visible link between the liberalization policy and the financial crisis. This result is very different from the exogenous school [7,11,13].

Through theoretical modeling, we demonstrate that the birth–death process is the proper model in population dynamics, which is capable of explaining all three observed features. The birth–death process originated in molecular dynamics in physics and has been introduced to describe an up–down process in the stock price movement [26–28]. We use the birth–death process as a unified model of linear (calm) and nonlinear (turbulent) markets. By means of moment expansion, we estimated the condition of the market breakdown, which is remarkably close to the real event. Unlike the model of heterogeneous agents, our population model of identical agents provides an alternate picture of animal spirits. Mass psychology is visualized by the rising and falling market tide that is measured by the net daily change rate.

Based on these findings, we get a new understanding of old conflicting thoughts. The so-called efficient market provides a simplifying linear picture of the calm market, whose higher moments are much smaller than the variance. The nonlinear turbulent market during the crisis resulted from the rise of high moments when the buy and sell pattern is remarkably nonlinear and asymmetric. There is strong evidence of endogenous instability, since the financial market is resilient under repeated cycles and crisis, which is characterized by the stable regime of the relative deviations [2,29,30]. Our picture greatly extends the scope of equilibrium models in finance, which can be considered as a special case of a calm market in our nonlinear model.

2. Theoretical models: the master equation and the birth–death process

2.1. The master equation

The master equation is widely used in physics, chemistry, biology and finance [18,31]. The change of the time-varying probability distribution is generated by the transitions from state x' to state x, minus the transitions from state x to state x'. The resulting master equation is given by the following partial differential equation:

$$\frac{\partial}{\partial t}P(x,t) = \int dx'[W(x|x',t)P(x',t) - W(x'|x,t)P(x,t)]. \tag{1}$$