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Hierarchical logistic equation to describe the dynamical behavior of penetration rates

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HIGHLIGHTS

- We unveil human communication behind the logistic equation as a diffusion model.
- The essential communication is to imitate adopters easily, which is so unnatural.
- The logistic equation is improved to the hierarchical logistic equation.
- Our model reflects a fact that there exist from the trend-conscious to the cautious.
- Our model can approximate the iPod sales data better than the logistic equation.

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ABSTRACT

We propose a hierarchical logistic equation as a model to describe the dynamical behavior of a penetration rate of a new product. In this model, a memory of how many *adopters* a *non-adopter* met is considered, where (non-)adopters mean people (not) possessing the product. This effect is not included in the logistic model. As an application, we apply this model to iPod unit sales data, and find that this model can approximate the data much better than the logistic equation. By using this model, we can guess a percentage of trend-cautious people in our society from the unit sales data.

1. Introduction

How does fashion diffuse in our society? Fashion spreads, although we do not aim for it to do so. New products that are somewhat of a curiosity at the beginning will become commonplace before we notice, and some of them may disappear from our life style. For instance, at the time when TV was invented, nobody imagined that a household would have TVs. Furthermore, at the time when CDs came out into the world, nobody dreamed of taking it out of the room and enjoying music. However, times have changed: We can enjoy music more easily by using MP3 players, such as the iPod. This phenomenon is similar to various changes of phase in matter which we cannot imagine from an interaction between atoms or molecules. That is to say, the human being is "the social atom" [1].

We want to unveil an essential human communication yielding a fashion diffusion in a society. In order to study this phenomenon precisely, indeed, we must consider other factors. For example, a decrease in the price of a product is one of the most important factors for it to spread into a society. TV could not diffuse at such a speed if the price did not drop from the original one. Because the manufacturing cost reduced, and then, the price lowered down to one at which people can get it, TV has become commonplace. We also should not ignore an increase in the functionality of a product. The reasons why MP3 players have been substituted for CD players are the better portability, the freedom from troublesome shuffling of CDs, and

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the larger number of songs. In other words, "the complexity ... is negatively related to its rate of adoption" [2]. In this paper, however, we do not think of these factors. That is to say, we suppose that the price and the functionality do not change.

In order to deal with this as a scientific problem, quantitative data indicating the extent of diffusion are necessary. Here, we shall employ a *penetration rate*. This has the following universality: Generally, a "slow advance in the beginning, followed by rapid and uniformly accelerated progress, followed again by progress that continues to slacken until it finally stops" [3]. This change in time is called the S-shaped curve. The logistic function, solution of the logistic equation, has often been employed to analyze the rates simply. The logistic equation can be represented by the following differential equation.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{a}{X} x(t) \left\{ X - x(t) \right\}. \tag{1}$$

When this equation is utilized for fashion diffusion, X and $x(t)$ mean the total population of a society and the number of those who possess a prevalent product, respectively. Furthermore, the parameter a is called the *coefficient of imitation* [4–6]. The explanation of a will be discussed later.

This solution is the logistic function:

$$x(t) = \frac{X}{1 + \left\{ \frac{X}{x(0)} - 1 \right\} \mathrm{e}^{-at}}. \tag{2}$$

If $x(0) < X$, then, $x(t)$ is in an interval from 0 to X and forms the S-shaped curve. Therefore, $x(t)/X$ can be employed for fitting a penetration rate since it does not extend beyond 1.

As is well known, the logistic equation was proposed as an equation describing a population growth with an upper limit by Verhulst [7–9]. However, the value of his study was not accepted in those days. In 1920, about a century later, Pearl and Reed rediscovered this equation while investigating the evolution of fly population [10]. Lotka also derived this equation as the model of population growth [11].

Griliches made the first adoption of the logistic equation for the dynamical behavior of innovation diffusion [12]. He analyzed the penetration rate of hybrid corn among farmers by the logistic function. After that, Mansfield explained the reason why the logistic equation can be used for the innovation diffusion mathematically [13]. On the other hand, Fisher and Pry utilized this for a substitution of a share of old and new products, e.g., margarine and butter [14]. This feature of the logistic equation in describing a replacement process is employed to predict urbanizations in some countries [15–17].

In this way, the employment of the logistic equation for innovation diffusion started, and then, this has been utilized for dynamical behavior of penetration rates for various products. In the past decade mobile phones [18–20], personal computers [21–24,4], electronics [21,25,26], energy technologies [27,28], information technologies [4] and oxygen-steel making process [5] were analyzed.

Thus, the change of many penetration rates in time can be described by the logistic function. But, what kind of human communication results in such a dynamical behavior? It is not a self-evident question. So, in this work, we shall distill the essential communication from the logistic equation. According to our study, it becomes clear that the logistic equation applied to penetration rates supposes the following human communication: Those who do not have a prevalent product start to possess it shortly after they meet people already possessing it, which will be verified by numerical simulation. Indeed, we cannot derive such a fact from the differential equation (1): From the re-formulated equation

$$\frac{\mathrm{d}x(t)/\mathrm{d}t}{X - x(t)} = a \frac{x(t)}{X}, \tag{3}$$

we can obtain nothing more than the following boring information, "The ratio for people who start to possess it afresh ($\mathrm{d}x(t)/\mathrm{d}t$) to those who do not have it ($X - x(t)$) is proportional to the ratio for people already possessing it ($x(t)$) to all the people (X)". So, we shall discretize the time and the space, and then, construct a lattice model. In doing so, we can unveil the above human communication from the model. This is done in Section 2.

Hence, a new question arises: Are we influenced by others easily?, which is the real starting point of this paper, since we know well empirically that there are not only trend-conscious people but also cautious people in our society. Therefore, we shall construct a new model supposing more natural human communication, which implies that we extend the logistic equation. This is discussed in Section 3.

Moreover, we adopt iPod unit sales as the real penetration data, and then, clarify that our model can describe the behavior much better than using the logistic equation in Section 4.

2. Penetration rate in an imitating group

In this section, we derive the logistic equation from a lattice model where people are influenced each other by four rules: We decompose the dynamics of the logistic equation into four elementary steps. In doing so, we can unveil essential human communications in the dynamical change of penetration rates described by the logistic function. In addition, we confirm this by numerical simulation.

Let us consider a group composed of N people. For this group, we shall apply the following rules: (i) At the beginning, some people have a product which will diffuse in this society. (ii) If those who do not possess the product yet (*non-adopters*) meet