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Cascade of failures in interdependent networks coupled by different type networks

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HIGHLIGHTS

- Different type coupled-networks are more vulnerable than the same type coupled-networks.
- If only lowly connected nodes were attacked, the system can still leads to a first order percolation phase transition.
- Different type coupled-networks are difficult to defend by strategies such as protecting the high degree nodes.
- If only the highly connected nodes were attacked, coupled scale free networks become more vulnerable than the others.

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ABSTRACT

Modern systems are mostly coupled together. Therefore, they should be modeled as interdependent networks. In this paper, the robustness of interdependent networks coupled with different type networks is studied in detail under both targeted and random attack. The critical fraction of nodes leading to a complete fragmentation of two interdependent networks is analyzed. Some findings are summarized as: (i) For random attack problem, the existence criteria for the giant component in interdependent networks coupled by two different type networks are quite different from those coupled by the same type networks. Different type coupled networks are more vulnerable than the same type coupled-networks. (ii) For targeted attack problem, if the highly connected nodes are protected and only the lowly connected nodes failed, the system leads to a first order percolation phase transition for different type coupled-networks, and a second transition for same type coupled-networks as well. The available result implies that different type coupled-networks are difficult to defend by strategies such as protecting the high degree nodes that can be useful to significantly improve robustness of the same type coupled-networks. (iii) For targeted attack problem, when the lowly connected nodes are protected and only the highly connected nodes failed, coupled scale free networks become more vulnerable than the others.

1. Introduction

In recent years, extensive efforts have been paid to study and understand the properties of complex networks. Most of the research have only concentrated on the limited case of a single, noninteracting network [1–8]. However, modern

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real networks are non-isolated, and they are coupled mostly together and therefore should be modeled as interdependent networks [9–16].

In interdependent networks, a fundamental property is that failure of nodes in one network may lead to failure of dependent nodes in other networks. This may happen recursively and can lead to a cascade of failures. From both analytical and numerical viewpoints, it has been shown that the robustness of two interdependent networks is significantly lower than that of a single network. The most dangerous vulnerability is hiding in many interdependencies across different networks [17–27].

In 2010, Buldyrev et al. [17] developed a theoretical framework for studying the process of cascading failures in interdependent networks caused by random initial failure of nodes. They found that a broader degree distribution could increase the vulnerability of interdependent networks to random failure in contrast to the behavior of a single network. In the real world, for more than two networks coupled together, Gao et al. [18] proposed a framework to study the robustness of a network of networks (NON). Parshani et al. [19] studied the coupling networks where not all the nodes of network A depend on network B. Li et al. [20] studied the cascading failures in a system composed of two interdependent square lattice networks. Liu et al. [21] found that first-order and second-order phase transitions can occur in the more general setting where no interdependent links were present. Shao et al. [28] studied the cascade of failures in two coupled network where multiple support-dependence relations are randomly built. For the other results with respect to the robustness of interdependent networks, please refer to Refs. [27,29,28,30,7,31,32].

Previous studies on two interdependent coupled networks are restricted in the condition that network A and network B are the same type networks. However, in the real world, this assumption may not be realistic, where the network might be coupled by two different type networks. These real-world networks also share some structural properties such as scale-free degree distribution, small-worldness. The dynamical behavior of these coupled-networks largely depends on their structural properties. Understanding the robustness of different type coupled-networks is one of the major challenges for interdependent networks. The primary goal in this paper is to compare the vulnerability conclusions that result from same type coupled-networks with those that result from a more realistic model by different type coupled-networks.

In this paper, the coupling between two N nodes networks A and B is considered. The N nodes in each network are connected to nodes in the other network by bidirectional dependency links with the following restrictions: (i) The connections within network A and network B are different; (ii) Each node in network A depends on one node from network B, and vice versa; (iii) If node N_{Ai} depends on node N_{Bi}, then node N_{Bi} depends on node N_{Ai}. Thus, a one-to-one correspondence between network A and network B is established. The functioning of a node in network A depends on the functioning of the corresponding node in network B, and vice versa [17–22]. We will show that when a critical fraction of the nodes in one network fails for such a model, the system undergoes a first order phase transition due to the recursive process of cascading failures. If the highly connected nodes are protected and only the lowly connected nodes fail, different type coupled-networks are significantly more vulnerable than the same type coupled-networks.

2. Random-attack problem in interdependent networks

In order to investigate the robustness of interdependent networks under random-attack on nodes, the following examples are performed:

(i) Erdős–Rényi coupled-networks (ER–ER), Watts–Strogatz coupled-networks (WS–WS), and ER–WS coupled networks,
(ii) Newman–Watts coupled-networks (NW–NW), scale free coupled-networks (SF–SF), and NW–SF coupled networks,
(iii) Watts–Strogatz coupled-networks (WS–WS), scale free coupled-networks (SF–SF), and WS–SF coupled networks.

For Erdős–Rényi network, Watts–Strogatz network, and regular network, we choose average degree as $k = 4$. The random of reconnected edges is $P_r = 0.3$ in WS small world networks. For scale-free network, we first choose three nodes with three edges, choosing two nodes randomly from existing nodes when a node was added, and add two edges to connect them. The random of added nodes is $P_a = 0 : 05$ in NW small world networks.

2.1. Random-attack problem in ER–ER, WS–WS and ER–WS networks

A fraction $1-p$ of the nodes of network A is removed, P_A and P_B are defined as the fraction of nodes belonging to the giant components of network A and network B, respectively. The iterative process of cascading failures is initiated by removing a fraction $1 - p$ of network A nodes and all the A edges that are connected to them. The remaining fraction of network A is p. Network A may break into clusters and the remaining functional part of network A therefore contains a fraction $\psi_1 = pP_A(p)$ of the network nodes [17–22]. Since network B depends on the nodes from network A, the number of nodes in network B that becomes nonfunctional is $1 - pP_A(p)$, the remaining fraction of network B is $\phi'_1 = pP_A(p)$, and the fraction of nodes in the giant component of network B is $\phi_1 = \phi'_1 P_B(\phi'_1)$. Let $\psi'_1 = p$, one can get

$$\begin{aligned}
&\psi'_1 = p, \qquad \psi_1 = \psi'_1 P_A(\psi'_1),\\
&\phi'_1 = pP_A(\psi'_1), \qquad \phi_1 = \phi'_1 P_B(\phi'_1),\\
&\psi'_2 = pP_B(\phi'_1), \qquad \psi_2 = \psi'_2 P_A(\psi'_2),\\
&\phi'_2 = pP_A(\psi'_2), \qquad \phi_2 = \phi'_2 P_B(\phi'_2), \ldots
\end{aligned} \tag{1}$$