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Solutions for a q-generalized Schrödinger equation of entangled interacting particles

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HIGHLIGHTS

- Solutions of a nonlinear Schrödinger equation.
- Time dependent wave functions.
- Free particles and Moshinsky-like potential.

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ABSTRACT

We report on the time dependent solutions of the q-generalized Schrödinger equation proposed by Nobre et al. (2011). Here we investigate the case of two free particles and also the case where two particles were subjected to a Moshinsky-like potential with time dependent coefficients. We work out analytical and numerical solutions for different values of the parameter q and also show that the usual Schrödinger equation is recovered in the limit of $q \to 1$. An intriguing behavior was observed for $q = 2$, where the wave function displays a ring-like shape, indicating a bind behavior of the particles. Differently from the results previously reported for the case of one particle, frozen states appear only for special combinations of the wave function parameters in case of $q = 3$.

1. Introduction

Tsallis statistics [1,2], based on the entropy

$$S_q = k\frac{1 - Tr\rho^q}{q-1}, \tag{1}$$

where k is a constant and q is a real parameter, has shown to be useful in the description of several systems where the additivity fails such as systems characterized by non-Markovian processes [3–6], nonergodic dynamics and long-range many-body interactions [7]. The nonadditive characteristic of S_q [8–10] has also brought new insights to several works in the

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area of complex systems [9,10] and contributed to diverse problems of quantum mechanics [11–21]. Recently, Nobre, Rego-Monteiro and Tsallis have proposed a nonlinear Schrödinger equation [22] (NRT equation) that admits soliton-like solutions (q-plane waves) with possible applications in several areas of physics, including nonlinear optics, superconductivity, plasma physics, and darkmatter. Since the applicability of linear equations in physics is usually restricted to idealized systems [22], the advance in the understanding of real systems makes necessary the use of nonlinear equations several times [23]. For a system with particle of mass m, the NRT equation can be written as

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\left(\frac{\Phi(\vec{x},t)}{\Phi_0}\right) = -\frac{1}{2-q}\frac{\hbar^2}{2m}\nabla^2\left(\frac{\Phi(\vec{x},t)}{\Phi_0}\right)^{2-q} + V(\vec{x},t)\left(\frac{\Phi(\vec{x},t)}{\Phi_0}\right)^{q}, \tag{2}$$

where the scaling constant Φ_0 guarantees the appropriate units for the different physical terms appearing in the equation, i is the imaginary unity and $\hbar$ is the Planck's constant. An interesting aspect related to this equation is the non-Markovian effect emerging from the presence of the nonlinearity, i.e., $q \neq 1$. A similar situation is found in the porous media equation [24], whose solutions may exhibit a short or a long tailed behavior and are connected to anomalous diffusion [25]. This intriguing equation has been the focus of intensive research in recent years [26–30]. For instance, a family of time dependent wave packet solutions have been also investigated in Ref. [12], a quasi-stationary solution for the Moshinsky model has been proposed in Ref. [28], and a classical field-theoretic approach has been considered in Refs. [29,30].

Here we study the problem of two particles in the context of the NRT equation focusing on time dependent solutions. We worked on the case for free particles (Section 2) and particles subjected to a time dependent Moshinsky-like potential (Section 3). For these situations, we observe that the nonlinearity creates an entanglement between the particles, which is not present in the usual scenario and it is essential for describing physical reality [31] inherent to quantum mechanics [32,33]. We summarize our results and conclusions in Section 4.

2. Free particles

Let us start our discussion by considering the q-generalized Schrödinger equation for a system of two particles in the absence of potential by assuming $m_1 = m_2 = m$ and defining $\psi = \Phi/\Phi_0$ without loss of generality. The time dependent Moshinsky-like potential is discussed in the next section. For this case the NRT equation takes the following form

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x_1,x_2,t) = -\frac{1}{2-q}\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)\psi(x_1,x_2,t)^{2-q}, \tag{3}$$

where x_1 and x_2 represent the particles coordinates.

Similarly to the situation worked out in Ref. [22, Eq. (3)] has the q-plane wave as solution

$$\psi(x_1,x_2,t) = \exp_q\left[\mathrm{i}(k_1x_1-\omega_1 t)\right]\otimes_q \exp_q\left[\mathrm{i}(k_2x_2-\omega_2 t)\right], \tag{4}$$

where $\otimes_q$ is the q-product [34]. This result shows that the nonlinearity of the Eq. (3) entangled the particles, leading us to different behaviors dependent on q (see Fig. 1). It is worth noting that $|\psi|^2$ obtained from the NRT equation does not has a probabilistic interpretation for $q \neq 1$. In order to recover that probabilistic interpretation, we could follow the procedures proposed by Rego-Monteiro and Nobre [29], and introduce a second field defined by means of an additional non-linear equation coupled with the NRT equation for $q \neq 1$. Furthermore, recovering the probabilistic interpretation of the NRT equation under the action of an spatial and time dependent potential (such as the Moshinsky-like potential worked out in the next section), in contrast with the free particle case, still is an open problem. Even in the free particle case, to obtain general time-dependent solutions for the NRT equation represents a cumbersome task due the intrinsic complexity of ψ that still need to be incorporated in this second equation for obtaining the second field. For this reason, we have focused our study only on the solutions for the NRT equation, that is, on ψ. Although this simplified approach lacks an exact probabilistic interpretation, for q not so far from one we could expected that $|\psi|^2$ will keep some information on the probability of find particles in a given position and time; also, this analysis can provides new insights and a better understating of the NRT equation.

In order to explicit solve Eq. (3), we use the q-Gaussian package as an ansatz,

$$\psi(x_1,x_2,t) = \left[1-(1-q)\left(a(t)\,x_1^2+b(t)\,x_2^2+c(t)\,x_1x_2+d(t)\right)\right]^{\frac{1}{1-q}} \tag{5}$$

where a, b, c, and d are appropriate time dependent coefficients to be found. The presence of a crossing term in Eq. (5) covers a more general scenario characterized by an initial situation of mixing between the variables x_1 and x_2.

By substituting (5) in (3), the left and right sides of the NRT equation becomes,

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x_1,x_2,t) = -\mathrm{i}\hbar\left(x_1^2a'(t)+x_2^2b'(t)+x_1x_2\,c'(t)+d'(t)\right)\psi(x_1,x_2,t)^q, \tag{6}$$

and

$$\begin{aligned} &-\frac{1}{2-q}\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)\psi(x_1,x_2,t)^{2-q}\Big/\left(\frac{\hbar^2}{2m}\psi(x_1,x_2,t)^q\right)\\ &= 2a(t)\left((q-1)b(t)\left(x_1^2+x_2^2\right)+(q-3)x_1x_2c(t)+(q-1)d(t)+1\right)+2(q-3)x_1^2a(t)^2\\ &\quad+2b(t)((q-3)x_1x_2c(t)+(q-1)d(t)+1)+2(q-3)x_2^2b(t)^2-c(t)^2\left(x_1^2+x_2^2\right). \end{aligned} \tag{7}$$