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Multiple-relaxation-time lattice Boltzmann modeling of incompressible flows in porous media

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HIGHLIGHTS

- A D2Q8 MRT-LB model is proposed for simulating incompressible porous flows at the REV scale.
- The generalized non-Darcy model is employed to describe the momentum transfer in porous media.
- The generalized Navier–Stokes equations can be recovered through the Chapman–Enskog analysis in the moment space.
- The MRT-LB model is demonstrated by numerical simulations of several typical two-dimensional porous flows.

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ABSTRACT

In this paper, a two-dimensional eight-velocity multiple-relaxation-time (MRT) lattice Boltzmann (LB) model is proposed for incompressible porous flows at the representative elementary volume scale based on the Brinkman–Forchheimer-extended Darcy model. In the model, the porosity is included into the pressure-based equilibrium moments, and the linear and nonlinear drag forces of the porous matrix are incorporated into the model by adding a forcing term to the MRT-LB equation in the moment space. Through the Chapman–Enskog analysis, the incompressible generalized Navier–Stokes equations can be recovered. Numerical simulations of several typical porous flows are carried out to validate the present MRT-LB model. It is found that the present numerical results agree well with the analytical solutions and/or other numerical results reported in the literature.

1. Introduction

Fluid flow and related transport phenomena in porous media have gained significant research interest due to the importance of related technological and industrial applications, which include contaminant transport in groundwater, crude oil exploration and extraction, radioactive waste management, hydrogeology and so on [1–3]. For incompressible flows in porous media at the representative elementary volume (REV) scale, the Darcy model, the Brinkman-extended Darcy model and the Forchheimer-extended Darcy model have been widely employed. However, the Darcy model and the two extended (Brinkman and Forchheimer) models have some intrinsic limitations in simulating porous flows [4,5]. In order to overcome the deficiencies of the above mentioned models, the Brinkman–Forchheimer-extended Darcy model (also called the generalized model) has been developed by several research groups [4–7]. In the generalized model, the viscous and inertial forces are incorporated into the momentum equation by using the local volume-averaging technique at the REV scale. The Darcy model and the two extended models can be regarded as the limiting cases of the generalized model. Due to the

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similarity with the Navier–Stokes equations, the generalized model can be used to simulate transient flows through porous media. Moreover, as reported by Vafai and Kim [8], numerical results based on the Brinkman–Forchheimer-extended Darcy formulation have been shown to be in good agreement with the experimental predictions. In the past several decades, various traditional numerical methods, such as the finite volume (FV) method, the finite difference (FD) method, and the finite element (FE) method, have been employed to study porous flows based on the generalized non-Darcy model.

The lattice Boltzmann (LB) method, as a bran-new mesoscopic numerical technique originates from the lattice-gas automata (LGA) method [9], has achieved significant success in modeling complex fluid flows and simulating complex physics in fluids [10–16]. Owing to its kinetic background, the LB method has some distinctive advantages over the traditional numerical methods (e.g., see Ref. [17]). Recently, the LB method has been successfully applied to simulate fluid flows in porous media at the REV scale [18–25]. In the REV scale method, the detailed geometric structure of the media is ignored, and the standard LB equation is modified by adding an additional term to account for the presence of the porous media. Therefore, LB method at the REV scale can be used for systems with large computational domain. It is worth mentioning that the REV LB method is very effective for simulating fluid flows in the region which is partially filled with a porous medium. As reported in Ref. [21], the discontinuity of the velocity-gradient at the porous medium/free-fluid interface can be well captured by the LB method without including the stress boundary condition into the simulation model.

To the best of our knowledge, most of the existing REV LB models [18–25] for incompressible porous flows employ the Bhatnagar–Gross–Krook (BGK) model (also called the single-relaxation-time model) [26] to represent the collision process. Although the BGK-LB model has become the most popular one in the LB community because of its extreme simplicity, there are several well-known criticisms on this model, such as the numerical instability at low values of viscosity [27–29] and the inaccuracy in treating boundary conditions [30]. Fortunately, these shortcomings of the BGK-LB model can be easily addressed by employing the multiple-relaxation-time (MRT) model proposed by d'Humières [31]. Hence, the aim of this paper is to develop a new MRT-LB model for incompressible porous flows at the REV scale based on the generalized model. In the model, a pressure-based MRT-LB equation with the eight-by-eight collision matrix [32] is constructed in the framework of the standard MRT-LB method. The remainder of this paper is organized as follows. In Section 2, the MRT-LB model for incompressible porous flows at the REV scale is presented in detail. In Section 3, numerical tests of the MRT-LB model are performed for the porous Poiseuille flow, porous Couette flow, lid-driven flow in a square porous cavity, and natural convection flow in a square porous cavity. Finally, a brief conclusion is made in Section 4.

2. MRT-LB model for incompressible flows in porous media

2.1. Macroscopic governing equations

The fluid flow is assumed to be two-dimensional, laminar and incompressible. For incompressible flows through porous media at the REV scale, the generalized model proposed by Nithiarasu et al. [4] is employed in the present study. The dimensional governing equations (generalized Navier–Stokes equations) of the generalized model can be written as follows:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\left(\frac{\mathbf{u}}{\phi}\right) = -\frac{1}{\rho_0}\nabla(\phi p) + \upsilon_e \nabla^2 \mathbf{u} + \mathbf{F}, \tag{2}$$

where ρ_0 is the average fluid density, $\mathbf{u}$ and p are the volume-averaged fluid velocity and pressure, respectively, ϕ is the porosity, and υ_e is the effective kinetic viscosity. $\mathbf{F} = (F_x, F_y)$ denotes the total body force induced by the porous matrix and other external forces, which can be expressed as [6,22]

$$\mathbf{F} = -\frac{\phi \upsilon}{K}\mathbf{u} - \frac{\phi F_\phi}{\sqrt{K}}|\mathbf{u}|\mathbf{u} + \phi \mathbf{a}, \tag{3}$$

where υ is the kinetic viscosity of the fluid, K is the permeability, F_ϕ is the geometric function, $\mathbf{a}$ is the body force due to an external force, and $|\mathbf{u}| = \sqrt{u_x^2 + u_y^2}$, in which u_x and u_y are the x- and y-components of the macroscopic velocity $\mathbf{u}$, respectively. Based on Ergun's relation [33], the geometric function F_ϕ and the permeability K of the porous media can be expressed as [34]

$$F_\phi = \frac{1.75}{\sqrt{150\phi^3}}, \qquad K = \frac{\phi^3 d_p^2}{150(1-\phi)^2}, \tag{4}$$

where d_p is the diameter of the solid particle. The flow governed by the generalized Navier–Stokes equations (1) and (2) are characterized by the porosity ϕ and several dimensionless parameters: the Darcy number Da, the viscosity ratio J, and the Reynolds number Re, which are defined as

$$\mathrm{Da} = \frac{K}{L^2}, \qquad J = \frac{\upsilon_e}{\upsilon}, \qquad \mathrm{Re} = \frac{LU}{\upsilon}, \tag{5}$$

where L and U are the characteristic length and velocity of the system, respectively.