Physica A 448 (2016) 154–162

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An efficient heuristic method for dynamic portfolio selection problem under transaction costs and uncertain conditions

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HIGHLIGHTS

- A dynamic modeling of portfolio selection problem is investigated.
- A heuristic method is proposed to the problem considering transaction costs and uncertain conditions.
- Results show the superiority of the proposed method.

ARTICLE INFO

Article history:
Received 19 April 2015
Received in revised form 27 October 2015
Available online 31 December 2015

Keywords:
Portfolio selection
Dynamic optimization
Transaction cost
Investment strategy

ABSTRACT

Selecting the optimal combination of assets in a portfolio is one of the most important decisions in investment management. As investment is a long term concept, looking into a portfolio optimization problem just in a single period may cause loss of some opportunities that could be exploited in a long term view. Hence, it is tried to extend the problem from single to multi-period model. We include trading costs and uncertain conditions to this model which made it more realistic and complex. Hence, we propose an efficient heuristic method to tackle this problem. The efficiency of the method is examined and compared with the results of the rolling single-period optimization and the buy and hold method which shows the superiority of the proposed method.

1. Introduction

Portfolio optimization is used to achieve one of the goals of asset management which is selecting a suitable portfolio for investors. There are several solution approaches to solve the portfolio optimization problems. These solution approaches have a broad range such as meta-heuristic algorithms [1–3], fuzzy approaches [4,5], robust optimization [6], empirical analysis [7,8] and so on. One of the numerical approaches used in the literature is dynamic approach which in fact is a good fit for the nature of investment.

In 1969, Merton proposed a model for the continuous time problem which enabled investors revise their portfolios in the course of investment and arrange the consumption and investment in different assets so as to achieve maximum utility of subsequent consumptions and the expected investment utility [9]. He used nonlinear partial differential equations to find the optimal portfolio. The main aim of that study was to control the exchange of risky assets which enables investors to invest a fixed ratio of their wealth at all times in risky assets. He used a logarithmic utility function, called CARA (Constant Absolute Risk Aversion) as the objective function. One important property of the logarithmic utility function is that the strategy (how to partition the total available wealth) in order to reach optimal terminal utility is independent of the actual value of wealth and the ratio of the fund that we invest in each stock mainly depends on the amount of changes that we

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http://dx.doi.org/10.1016/j.physa.2015.12.048
0378-4371/

expect in that stock. However, one of the constraints on Merton's work was the lack of information on trading costs which led to over-simplification of the problem. Since 1970 various models covering trading costs have been proposed. Each of these models has its own specific computational complexity as well as pros and cons. In sum, the pioneers of dynamic optimization were Merton [10] and Samuelson [11]. However, the use of those methods was limited to constrained problems and specific priorities of investors; some examples of which can be found in the studies by Kim and Omberg [12], Liu[13], and Wachter [14].

In recent studies of this issue, the proposed numerical and approximation methods can be used to introduce more realistic features into portfolio optimization models. Examples of such studies include, Brennan et al. [15], numerically solved partial differential equations as a part of a dynamic optimization process. Kogan and Uppal [16] developed objective functions with different methods and managed to solve this problem using analytical methods. The other step toward simplification of the problem was discretization of problem space. This step enables researchers to use methods such as the following for assessing and solving problems: quadrature differential [17], simulation [18], binomial discretization [19], non-parametric regression [20], and dynamic optimization.

The main idea of dynamic optimization is simple and it says that in order to solve a problem it is necessary to solve different parts of the problem separately and then combine the solutions of sub-problems to achieve an overall solution to the whole problem. In more simplified methods, a large number of sub-problems are created and each of them is solved many times. However, in this method problems are solved only once and the overlap between subsystems is used to prevent solving the problem for the second time. By this approach the number of calculations will be reduced. The solution to each subsystem or sub-problem is stored to be used in the next step where the overall solution to the problem is calculated. This method is highly useful when the number of sub-problems increases exponentially [21].

A few studies have tried to solve the portfolio dynamic optimization problem based on trading costs and with several risky assets. Muthuraman and Kumar [22] solved this problem by updating the boundaries of the non-trading region. They solved Merton's model for several risky assets in a multidimensional state. In addition, Akian and Sulem [23] also studied a numerical method for maximizing growth rate with several risky assets based on trading costs. In 2010, Lynch and Tan [24] proposed a relatively simple method which was based on relative trading costs and dynamic planning.

In general, dynamic optimization is bounded with mathematical complexities which behaves exponentially as the problem size increases. However, few researches solved multidimensional equations of portfolio selection using partial differential equations [25]. Bielecki and Pliska [26] developed a continuous model of portfolio selection. The distinctive advantage of their model was considering the price of securities and changed depending on economic factors. Also, Cai et al. [27] and Brown and Smith [28] investigated the dynamic portfolio optimization problem.

As mentioned previously, several innovative methods have been developed for stating and solving problems through dynamic planning. Each of these methods can yield somewhat realistic results based on their assumptions, constraints and computational complexities. Therefore, they may converge to different results.

In this paper, it is tried to propose an efficient method to solve these problems so as to achieve more realistic results and avoid complex mathematical equations and approximations. More accurately, in this paper, a model is presented for practical multi-period optimization of portfolio and for selecting a suitable investment strategy for a specific period of time. In this research, the discrete time state is used with a dynamic optimization method. Therefore, the proposed model is capable of determining investment strategies for N revision points during the overall investment period (T) by considering trading costs so as to provide the optimal utility to the investor. In this regard, introduction of trading costs to the model is a challenge that is tried to be solved with a different approach. Some practical solutions are proposed to this problem as well.

The rest of our work is organized as follows. In the next section we describe the research methodology and present our proposed algorithm. In Section 3, the model validation is examined by applying two examples. Finally, conclusion comes in Section 4.

2. Research methodology

The multi-period optimization problem discussed in this paper can be written as follows [27].

$$V_t\left(W_t, x_t\right) = \max_{\delta_t} E\left\{W_{t+1}^{1-\gamma} \cdot g_{t+1}\left(x_{t+1}\right)\right\} \tag{1}$$

s.t.

$$\begin{aligned}
&\Pi_{t+1} = \mathrm{e}^T X_{t+1} + R_f\left(1 - \mathrm{e}^T x_t - y_t\right),\\
&S_{t+1,i} = R_i\left(x_{t,i} + \delta_{t,i}\right),\\
&y_t = \mathrm{e}^T\left(\delta_t + \boldsymbol{\tau}\left|\boldsymbol{\delta_t}\right|\right),\\
&W_{t+1} = \Pi_{t+1} \cdot W_t,\\
&\delta_t = x_{t+1,i} - x_{t,i},\\
&x_{t+1,i} = S_{t+1,i}/\Pi_{t+1},\\
&t = 0, 1, \ldots, T-1; \quad i = 1, \ldots, m
\end{aligned}$$