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Heterogeneity of link weight and the evolution of cooperation

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HIGHLIGHTS

- We confirm that the heterogeneity of link weight promotes evolution of cooperation.
- We identify two key mechanisms whereby heterogeneity enhances cooperation.
- The identified mechanisms are those for the spread/maintenance of cooperation.
- We derive the conditions under which the above-mentioned mechanisms can work.

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ABSTRACT

In this paper, we investigate the effect of *heterogeneity of link weight*, heterogeneity of the frequency or amount of interactions among individuals, on the evolution of cooperation. Based on an analysis of the evolutionary prisoner's dilemma game on a weighted one-dimensional lattice network with *intra-individual heterogeneity*, we confirm that moderate level of link-weight heterogeneity can facilitate cooperation. Furthermore, we identify two key mechanisms by which link-weight heterogeneity promotes the evolution of cooperation: mechanisms for spread and maintenance of cooperation. We also derive the corresponding conditions under which the mechanisms can work through evolutionary dynamics.

1. Introduction

The evolution of cooperation, which plays a key role in natural and social systems, has attracted much interest in diverse academic fields, including biology, sociology, and economics [1,2]. The prisoner's dilemma (PD) is often used to study the evolution of cooperation in a population consisting of selfish individuals [3,4]. In the PD game, two individuals simultaneously decide to cooperate or defect. A payoff matrix of the PD game is given in Table 1.

If either individual wishes to maximize his/her personal profit in this game, he/she will choose to defect regardless of the opponent's decision, despite mutual cooperation being better than mutual defection for both individuals. According to the evolutionary dynamics of the PD game where an individual is paired with a randomly chosen opponent in a well-mixed population, cooperators become extinct whereas defectors eventually dominate in the population [5].

However, in a dilemma situation in the real world, we often see that altruistic behaviors exist among unrelated individuals. Nowak [6] proposed *five rules* as the mechanisms enabling the evolution of altruism: kin selection [7], direct reciprocity [4,8], indirect reciprocity [9,10], network reciprocity [11–19], and group selection [20].

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Table 1
Payoff matrix for the prisoner's dilemma (PD) game. In this game, two individuals decide simultaneously to cooperate or defect. Mutual cooperation provides them both with a payoff R, whereas mutual defection results in a payoff P. If one individual cooperates and the other defects, the former obtains a payoff T, and the latter a payoff S. These values are assumed to satisfy the conditions $T > R > P > S$ and $2R > S + T$.

	Cooperation	Defection
Cooperation	R, R	S, T
Defection	T, S	P, P

In this study, we focus on *network reciprocity*, which is a mechanism pioneered by Nowak and May [11,12] that enables the evolution of cooperation when each individual is likely to interact repeatedly with a fixed subset of the population only. In Nowak and May's model, individuals are placed on nodes in a two-dimensional lattice and play the PD game repeatedly with their directly connected neighbors only. The authors show that the spatial constraint of interactions among individuals in the lattice network can facilitate the evolution of cooperation. Although Nowak and May's model assumes that the population has a simple network structure, that is, a two-dimensional lattice, it has recently been shown that many real-world networks are identified as *complex networks*. Well-known examples of complex networks are the small-world network [21] and the scale-free network [22], in which *the number of links* (degree) that each individual has differs. Recently, it has been confirmed by Santos and Pacheco [13] that *heterogeneity of the number of links* in complex networks can enhance the evolution of cooperation. There have been following studies that investigate the evolution of cooperation on networks with heterogeneous number of links [14,15]. This heterogeneity is also known to contribute to the efficiency of collective action [23,24]. Additionally, it has been shown that the mixing pattern of link degree can affect the emergence of cooperation [16]. See Refs. [17,18] for detailed reviews of evolutionary and coevolutionary games on graphs. Also see Ref. [19] for a thorough survey of the evolutionary dynamics of group interactions on various types of structured populations.

The aforementioned studies, however, assume that individuals interact with one another with the same frequency or amount; that is, all the link weights between individuals in the society are identical. On the contrary, individuals in real-world networks, such as scientific collaboration networks, phone call networks, email networks, and airport transportation networks, have heterogeneous intentions in their relationships [25–27]. There is substantial interest among researchers in knowing how heterogeneity of the strength of relationships (that is, link weight) among individuals influences human behavioral traits (e.g. sociological studies such as Refs. [28–30]).

In particular, researchers have recently investigated whether the heterogeneity of link weight between individuals promotes the evolution of cooperation. For example, Du et al. [31] constructed a simulation model in which individuals are placed on a node in a scale-free network and connected to other individuals with heterogeneous link weights. In their model, individuals interact more frequently with neighbors connected by links with large weights and less frequently with those connected by links with small weights. Du et al. found that cooperative behavior can be more facilitated when the link weights shared by individuals are heterogeneous rather than homogeneous. Note that, in their model, interaction networks have two kinds of heterogeneity: heterogeneity of the *number of links* and that of *link weight*. Note also that each link weight is determined according to the number of links of individuals; that is, link weight is a function of the degrees of the two individuals at either side of the focal link. Therefore, in the Du et al. model it is difficult to ascertain which factor enhances cooperation: *heterogeneity of the number of links* or *heterogeneity of link weight*.

Additionally, Ma et al. [32] employed a two-dimensional square lattice with individuals placed on its nodes. In their model, individuals play the PD game with their immediate neighbors connected by links with heterogeneous link weights. Ma et al. arranged three populations, where the link weights in the population follow either power-law, exponential, or uniform distribution patterns. They confirmed that a network with a power-law distribution of link weights better facilitates the evolution of cooperation than one with link weights conforming to one of the other two probability distributions.

Because a two-dimensional square lattice is used in their model, each individual has the same number of links (i.e., four). Thus, their result clearly shows that *heterogeneity of link weight* can bring about a cooperative state even without *heterogeneity of the number of links*. However, in their model, the sum of link weights of an individual, which we call the *link-weight amount* of the individual, differs from those of others. That is, not only each of the links possessed by an individual can have a different weight, but the individual can also have a different link-weight amount from other individuals. We call the former *intra-individual heterogeneity* and the latter *inter-individual heterogeneity*.

When *inter-individual heterogeneity* exists, some individuals play the PD game more frequently than others (link-weight amount is heterogeneous among individuals). That is, there is *heterogeneity of the interactions among individuals*. It has already been shown [13–15] that heterogeneity of interactions among individuals due to *heterogeneity of the number of links among individuals* and not to *inter-individual heterogeneity* can facilitate the evolution of cooperation.

Fig. 1 shows three examples of a one-dimensional lattice having, respectively, *intra-individual heterogeneity*, *inter-individual heterogeneity*, and *heterogeneity of the number of links*. Fig. 1(a) shows an example of *inter-individual heterogeneity*, where individuals on the left-hand side have large link-weight amounts and those on the right-hand side have small link-weight amounts. In this case, individuals on the left-hand side interact more frequently with others than those on the right-hand side; that is, there is heterogeneity of interactions between individuals. *Heterogeneity of the number of links*