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Fluctuation behaviors of financial return volatility duration

Hongli Niu *, Jun Wang, Yunfan Lu

Institute of Financial Mathematics and Financial Engineering, School of Science, Beijing Jiaotong University, Beijing 100044, China

HIGHLIGHTS

- A new concept of the durations of daily financial return volatility is introduced.
- Probability distributions, memory effects and multifractal properties of volatility duration series are investigated.
- The proposed concept of the duration is a meaningful and beneficial trial for the stock volatility series analysis.
- Empirical research of the durations is performed for world's seven representative stock indices.

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ABSTRACT

It is of significantly crucial to understand the return volatility of financial markets because it helps to quantify the investment risk, optimize the portfolio, and provide a key input of option pricing models. The characteristics of isolated high volatility events above certain threshold in price fluctuations and the distributions of return intervals between these events arouse great interest in financial research. In the present work, we introduce a new concept of daily return volatility duration, which is defined as the shortest passage time when the future volatility intensity is above or below the current volatility intensity (without predefining a threshold). The statistical properties of the daily return volatility durations for seven representative stock indices from the world financial markets are investigated. Some useful and interesting empirical results of these volatility duration series about the probability distributions, memory effects and multifractal properties are obtained. These results also show that the proposed stock volatility series analysis is a meaningful and beneficial trial.

1. Introduction

The study on the statistical properties of price fluctuations is very important to understand and model the dynamic behaviors of financial markets. It has also long been a focus of economics research. The stock price variations usually exhibit some universal statistical features such as fat tails phenomenon, power law of logarithmic returns and volumes, volatility clustering, multifractality of volatility, etc. [1–20]. In particular, understanding the return volatility is of particularly crucial for the asset price theory as well as the practical financial management because it contributes to quantifying the risk, optimizing the portfolio [21,22], and providing a key input of option pricing models that are based on the estimation of the volatility of the asset [23,24,22]. In recent years, the waiting time between consecutive events has aroused a great deal of attention, for instance, the intertrade duration between two consecutive trades [25,26], the duration time that the price or volatility keeps below or above its initial value [27,28], and the waiting time that the price return first exceeds a predefined level [29–31]. Particularly, many literatures have studied the volatility return intervals or the reoccurrence times τ between

* Corresponding author. Tel.: +86 10 51466946.
E-mail address: niuhonglibjtu@yeah.net (H. Niu).

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successive events above or below some threshold Q (which are usually referred to rare or extreme events), and the return intervals exhibit some interesting statistical characteristics [32,4,33–35]. For example, Yamasaki et al. [35] found that the distribution function $P_Q(\tau)$ scales with the mean return interval $\bar{\tau}$ as $P_Q(\tau) = \bar{\tau}^{-1}f(\tau/\bar{\tau})$, and $f(x)$ is consistent with a power-law form. The return intervals also display strong memory effects, which is related to the long-term correlation that is well-known to be present in the volatility. Bogachev and Bunde [32] showed that for multifractal data sets from the multiplicative random cascades (MRC) and the multifractal random walks (MRW) models, the probability density functions (PDFs) and the autocorrelation function as well as the conditional return period of the interoccurrence times (between events above some threshold) are governed by power laws with exponents that depend explicitly on the considered threshold. Xie et al. [34] investigated the extreme value statistics and recurrence intervals of the volatilities for NYMEX energy future.

Inspired by the extant return interval analysis, we introduce a new definition of the duration of daily stock volatility consistently above or below a given data point in the volatility series. However, it is different in the way of choosing threshold, since the threshold is not predetermined but is changing with the volatility intensity along the series. If the volatility intensity of next day is larger (or smaller) than the current one, we say the current volatility is locally rising (or falling). The volatility duration is then defined as the shortest passage time when the future volatility intensity of stock prices is below (or above) the current volatility intensity, see Section 3. Through the definition, we somehow build a link between volatility intensity and duration, and expect to gain some insights by this linkage. Actually, this approach also shares much similarity with the ideas embodied in intensity–duration–frequency relationship [36], which is a method widely used in description of data records of precipitation, climate, flood and wave etc. What is more, this concept is somewhat slightly like the idea behind two simple quantities related to extreme statistics: the mean exit time (MET) which is the mean time when the random process leaves for the first time a given interval, and the mean first-passage time (MFPT) which is the average time when the random process reaches for the first time some pre-assigned value. In Ref. [37], the authors studied the mean escape (exit) time in a market model with a cubic nonlinearity coupled with a stochastic volatility described by the Cox–Ingersoll–Ross equation. It was shown that (in a proper parameter region) the probability density function of the escape times of the returns fits very well, which are obtained from the experimental data extracted by the real market. In Ref. [38], the theory of continuous time random walks was applied to study the extreme events, and shown that these extreme times obey an integral equation which depends on the jump distribution and the mean waiting time. To a small extent, the lower bound of a given interval is similar to the current volatility intensity, and the upper bound is like the first volatility intensity larger than the current one in this proposed duration concept. But in volatility duration, the so-called fixed value is taken as the current volatility value which is always different. The motivation for proposing the volatility duration is as follows. Apart from caring the future's volatility intensity above or below a predefined large value, one trader would more like to know the shortest passage time (duration) that takes the future's volatility intensity to go beyond or below the *current volatility* intensity, which also means that trader is more likely to make decisions by taking the current volatility as a reference. If it is locally falling (compared with the "tomorrow" volatility), one is interested in the minimum time that it takes for the future's volatility intensity to exceed the current value, and this minimum time is referred as the volatility duration and vice verse. We afterwards analyze the statistical properties of the proposed volatility durations of seven representative stock indices from the world financial markets and the Gaussian data. Three aspects are mainly focused on: (i) The probability density function (PDF) of the volatility durations and the scaling behaviors are observed. (ii) The memory effects of the volatility durations are investigated by autocorrelation approach, the detrended fluctuation analysis (DFA) [33,20] and the detrending moving average (DMA) [39] techniques respectively. (iii) Their multifractal properties are studied by using the multifractal detrended fluctuation analysis (MF-DFA) method, which is famous for quantifying the multifractality nature concealed in the nonstationary time series [5,8,14–17].

2. Data sets

To investigate the volatility duration of financial time series, the data employed in this study consists of daily closing prices of seven important and representative stock indices from the world financial markets. That is, two American indices: NASDAQ Composite Index (IXIC) from 1 Dec. 1999 to 2 Dec. 2014 with 3775 data points, and Standard & Poor's 500 Index (S&P500) from 3 Jan. 2000 to 2 Dec. 2014 with 3753 data points; two European indices: Deutscher Aktienindex Index (DAX) from 10 Feb. 2000 to 2 Dec. 2014 with data length of 3778, and Financial Times Stock Exchange (FTSE) 100 Index from 1 Aug. 2000 to 2 Dec. 2014 with data length of 3736; finally three Asian indices: Nikkei 225 Index (N225) covers the time period from 1 Nov. 1999 to 2 Dec. 2014 with 3714 data points, Shanghai Stock Exchange Composite Index (SSE) covers the time period from 3 Jan. 2000 to 5 Nov. 2014 with 3773 data points, and Shenzhen Stock Exchange Component Index (SZSE) is from 1 Dec. 1999 to 5 Nov. 2014, which accounts to 3755 data points. The raw data sets are downloaded from the web http://finance.yahoo.com. Only the time series during the trading hours are adopted and those non-trading time series are regarded as frozen periods.

3. Concept of financial volatility duration

In this section, we introduce a new definition of volatility duration for stock returns. We begin by generating a volatility duration series $\{D(t), t = 1, 2, \ldots, T\}$ from a normalized volatility time series. Denote $P(t)$ the price at time t. The volatility