Physica A 448 (2016) 172–180

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Using dynamic mode decomposition to extract cyclic behavior in the stock market

Jia-Chen Hua [a,c,*], Sukesh Roy [b], Joseph L. McCauley [a], Gemunu H. Gunaratne [a]

[a] Department of Physics, University of Houston, Houston, TX 77204, United States
[b] Spectral Energies, LLC, Dayton, OH 45431, United States
[c] School of Electrical and Information Engineering, University of Sydney, NSW 2006, Australia

HIGHLIGHTS

- Application of a new technique (Koopman mode analysis) to stock valuations.
- Revealed four, hereto unknown, cyclic variations in stock market data.
- One of these has a period of 1 year which represents seasonal variations.
- Differentiated robust, repeatable features from noise and irregular characteristics.
- Opens up new possibilities of applying Koopman mode analysis in other research areas.

ARTICLE INFO

Article history:
Received 26 May 2015
Received in revised form 25 October 2015
Available online 29 December 2015

Keywords:
Econophysics
Business cycle
Stock markets
Koopman operator
Dynamic mode decomposition
Complex system

ABSTRACT

The presence of cyclic expansions and contractions in the economy has been known for over a century. The work reported here searches for similar cyclic behavior in stock valuations. The variations are subtle and can only be extracted through analysis of price variations of a large number of stocks. Koopman mode analysis is a natural approach to establish such collective oscillatory behavior. The difficulty is that even non-cyclic and stochastic constituents of a finite data set may be interpreted as a sum of periodic motions. However, deconvolution of these irregular dynamical facets may be expected to be *non-robust*, *i.e.*, to depend on specific data set. We propose an approach to differentiate robust and non-robust features in a time series; it is based on identifying robust features with *reproducible* Koopman modes, *i.e.*, those that persist between distinct sub-groupings of the data. Our analysis of stock data discovered four reproducible modes, one of which has period close to the number of trading days/year. To the best of our knowledge these cycles were not reported previously. It is particularly interesting that the cyclic behaviors persisted through the great recession even though phase relationships between stocks within the modes evolved in the intervening period.

1. Introduction

The existence of cyclic expansions and contractions in economic activity on a scale of 5–50 years has been long established [1]. Their study can be traced back to 1800’s [2] and an early report of a cycle (of duration 7–11 years) was made by the French statistician Clement Juglar in 1860 [3,4]. Subsequent studies established evidence for additional cycles

* Corresponding author at: School of Electrical and Information Engineering, University of Sydney, NSW 2006, Australia.
E-mail address: jchua@uh.edu (J.-C. Hua).

http://dx.doi.org/10.1016/j.physa.2015.12.059
0378-4371/

including the Kitchin inventory cycle [5] of period 3–5 years, the Kuznets swing of duration of 15–25 years [6], and the Kondratiev wave (or long technological cycle) of period 45–60 years [7]. Much less is known about cyclic behavior in financial markets. The goal of the work reported here is to utilize daily variations in stock prices to search for such cyclic activity.

Speculations abound on precursors to and derivatives of economic changes; however, due to their qualitative nature, establishing such causalities is non-trivial. One quantifiable measure of activities related to the economy or to financial market activity is the valuation of stocks. The valuation of a stock emerges from a series of complex interactions between participants such as investors, traders, and banks under constrains imposed by regulations and exchange rules, and reflect their collective estimation of the performance of the entity and the overall economy.

Variation of the price of a single stock depends on a multitude of factors, a majority of which is specific to the commodity. Hence, the search for changes in the overall financial market using a single index or a small set of indices is unlikely to bear fruit. Indeed, it has been found that a very large number of wavelet modes are required to decompose financial market dynamics [8,9]. A collective analysis of a large set of stocks is more likely to yield global market dynamics. Since little is known *a-priori* on relationships between individual stocks, empirical basis expansions, such as proper orthogonal decomposition (POD) [10–14], is a natural approach to follow. In fact, POD and its variants, such as *singular spectrum analysis* (SSA) and its extension *Multivariate SSA* (M-SSA), have proved effective in the study of financial markets [15–20]. SSA uses a short length sliding window to filter the original time series and constructs the trajectory matrix whose columns are lagged vectors with same length as the sliding window; M-SSA constructs a stacked matrix of multivariate time series from a set of individual times series. SSA and M-SSA perform singular value decomposition on the trajectory matrix, implicitly assuming that the original time series is stationary for the entire duration.

Koopman decomposition [21–24] is a natural approach to search for collective oscillatory behavior. It is based on *Koopman operator theory* [25,22,24], which generalizes eigendecomposition to nonlinear systems. Koopman modes are generalizations of normal modes [24] and each mode represents a global collective motion in (the assumed) market dynamics. Importantly, spectral properties of market dynamics will be contained in the spectrum of the Koopman operator [22,24]. A fast algorithm proposed by Schmid [25] and referred to as *dynamic mode decomposition* (DMD), can be used for computing approximately (a subset of) the Koopman spectrum from the time-series of valuations of a collection of stocks.

Each Koopman mode is associated with a complex eigenvalue whose real and imaginary parts represent the growth rate and frequency of the mode. Interestingly, Koopman eigenvalues may be used to differentiate robust characteristics of the underlying dynamics from those that depend on the specific data set [26]. The starting point is a search for *reproducible* Koopman modes, *i.e.*, those that persist in different sub-groupings of the time-series. The differentiation is based the following conjecture: *robust characteristics of the market dynamics can be associated with reproducible Koopman modes while non-robust features can be associated with non-reproducible modes.* Only robust features are expected to be useful in an analysis of financial market dynamics.

The methodology is summarized in Fig. 1. Our studies are conducted using valuations of the 567 stocks, each of whose market capitalization exceeded 7.5 billion dollars as of October 2014 and price history can be traced back to November 2002. The total market capitalization of these stocks is over 23 trillion dollars as of October 2014. The daily adjusted close prices were retrieved from "Yahoo! finance" (http://finance.yahoo.com/). Additional details of the filtering and choice of stocks, as well as data retrieval, is given in the Supplementary Materials (see Appendix A). The 567 stocks are grouped according to sector and industry, and placed in a two-dimensional array to aid visualization. Dynamic mode decomposition of the data yield a large number of modes, the spectrum of the most significant of them is shown in Fig. 1(a). The data is then partitioned into different subgroups, *e.g.*, the time-series for even and odd numbered trading days. We find only four common dynamic modes, which (in order of decreasing contributions to the data) are associated with periods 250 ± 4, 108 ± 2, 91 ± 1, and 76 ± 2 trading days respectively. To the best of our knowledge, these cycles shown in Fig. 1(b), have not been reported before. The corresponding eigenfunctions capture the phase relationships between different stocks in the sample; for example, one stock (or a sector) may react to economic changes immediately, while others may lag the changes by a few weeks. Fig. 1(c) shows the real part of one of the reproducible eigenfunctions. Finally, Fig. 1(d) illustrates the daily variations in the valuations of Intel Corporation in one reproducible mode.

2. Methods and results

2.1. Koopman decomposition

Denote the state of the market by a vector $\mathbf{z}$, whose constituents may include factors like the interest rate, expectations of economic growth, rate of unemployment, etc.; at the outset we recognize the precise form of $\mathbf{z}$ is unknown. Next, assume that market dynamics can be expressed as $\dot{\mathbf{z}} = \mathcal{F}(\mathbf{z})$; once again, its form is unknown. What we have available is "secondary" data, namely stock prices, which we expect to reflect changes in $\mathbf{z}$. Let us index the stocks in a pre-specified array $\mathbf{x}$, and express their values at time t through a field $u[\mathbf{z}](\mathbf{x}, t)$. When the context is clear, we will simplify the notation by eliminating the state $\mathbf{z}$ from this expression.

Koopman analysis [21–24] of the system is conducted on the "observables" $u[\mathbf{z}](\mathbf{x}, t)$. Suppose the initial state $\mathbf{z}_0$ of the market evolves under $\mathcal{F}$ to a state $\mathbf{z}_t$ at time t. During this time interval, an initial function $u[\mathbf{z}_0](\mathbf{x}, t = 0)$ evolves to a function $u[\mathbf{z}_t](\mathbf{x}, t)$. The transformation between the functions is given by the Koopman (or composition) operator, defined as

$$U^t : u[\mathbf{z}_0](\mathbf{x}, t = 0) \to u[\mathbf{z}_t](\mathbf{x}, t). \quad (1)$$