Physica A 448 (2016) 10–20

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Entanglement in a four qubit J_1–J_2 Heisenberg XXZ system with Dzialoshinskii–Moriya interaction

Alev Şahintaş, Cenk Akyüz *

Department of Physics, Adnan Menderes University, Aytepe, 09100, Aydın, Turkey

HIGHLIGHTS

- We consider both the nearest and the next nearest neighboring interactions.
- Dzialoshinskii–Moriya (DM) interaction enhances the nearest neighboring entanglement.
- Frustration augments the entanglement between the next nearest neighboring qubits.
- Contribution of frustration on entanglement is more effective than DM interaction.

ARTICLE INFO

Article history:
Received 26 March 2015
Received in revised form 27 November 2015
Available online 29 December 2015

Keywords:
Concurrence
DM interaction
J_1–J_2 Heisenberg XXZ model

ABSTRACT

In this study, we investigate the entanglement properties of a four qubit anisotropic Heisenberg XXZ system which has the nearest neighboring (NN), the next nearest neighboring (NNN) and Dzialoshinskii–Moriya (DM) interactions. Calculations of the ground state and thermal entanglement are carried out in terms of concurrence for selected ranges of control parameters such as DM interaction, anisotropy and frustration. From the results obtained, we see that DM interaction and the frustration play an active role on the ground state entanglement between NN and NNN qubits, respectively. We also see that frustration parameter α exhibits positive effects on the thermal entanglement especially between NNN qubits. Consequently, we show that effective control of entanglement can be obtained by employing competing effects of the control parameters in this general Heisenberg model which is constructed by considering not only NN interaction but also NNN and DM interactions.

1. Introduction

Entanglement or non-local correlations among systems which have unfactorizable total wave functions is one of the most important notion of quantum mechanics [1–3]. In the old quantum theory era [1,2], entanglement was considered as a mystery, however, nowadays we see it as a valuable resource for secure and fast quantum information processing [4], such as quantum teleportation [5,6], superdense coding [7], and quantum key distribution [8].

Low dimensional systems have emerged as an important research field since the beginning of the quantum mechanics. Although this importance partially arises from the systems can be achieved experimentally, it is mainly based on constituting the simple models such as Ising and Heisenberg models in one dimension for investigation of many body systems. Besides explaining certain physical features of crystal structures, these models are employed for modeling quantum computers [9], as well as quantum dots [10,11], nuclear spin [12], and optical lattices [13]. Furthermore, spin chains in solid state systems are

* Corresponding author. Tel.: +90 2562128498; fax: +90 2562135379.
E-mail address: cenk.akyuz@adu.edu.tr (C. Akyüz).

http://dx.doi.org/10.1016/j.physa.2015.12.069
0378-4371/

typical models due to their entanglement properties and realization of entanglement in them is very important. Therefore, investigation of the effects of externally controllable parameters, such as temperature, magnetic field, and anisotropy on entanglement is crucial. To this end, the entanglement analyses are carried out in Ising [14,15] and Heisenberg models [16–24]. A simple generalization of Heisenberg model so-called Majumdar–Ghosh or J_1–J_2 Heisenberg model is constructed by considering not only the nearest neighboring (NN) but also the next nearest neighboring (NNN) interactions [25,26]. Inserting a NNN interaction to system leads a frustration with respect to the sign of this interaction. Although these kind of interactions are commonly regarded as theoretical, they occur in some quasi-one-dimensional compounds, such as Cu–O [27], $CuGeO_3$ [28], and NaV_2O_5 [29]. Since frustration in spin systems is expected to lead to remarkable phases and exhibits reach entanglement properties, very recently investigation of entanglement in several Heisenberg models which have both NN and NNN interactions have been considered [30–37].

Besides spin–spin interaction, in Heisenberg spin systems, there is an anisotropic antisymmetric exchange interaction which arises from extension of the Anderson superexchange interaction theory by including the spin–orbit coupling effect [38]. This is known as Dzialoshinskii–Moriya (DM) interaction and it leads to a term in the Hamiltonian [38–41], equal to

$$\vec{D} \cdot (\vec{S}_1 \times \vec{S}_2). \tag{1}$$

Furthermore, many works in which spin models with DM interactions are taken into account could have realistic applications [42,43]. For instance, DM interaction is represented in certain quasi-one-dimensional magnets [42] and could be used to describe the compound $RbCoCl_3.2H_2O$ [44]. Thus, it should be noted that DM interaction is appreciable to achieve the spin-based realistic quantum computers [45]. Contrary to Heisenberg interaction which would align the neighboring spins parallel or antiparallel according to the system is ferromagnetic or antiferromagnetic, respectively, DM interaction would align them perpendicular to one another. Since such spin alignments are possible to increase entanglement, it turns out that DM interaction has a significant effect on entanglement properties [46–52].

In this paper, we investigate the ground state entanglement and thermal entanglement in a four qubit anisotropic J_1–J_2 Heisenberg XXZ spin system with DM interaction. The aim of this investigation is to point out how to strength entanglement in case when it already exists or to create entanglement in case when it does not exist by using a simple generalization of bilinear spin–spin interaction of the Heisenberg form and DM interaction. It is found that DM interaction and frustration play an effective role on the ground state entanglement between NN and NNN qubits, respectively. We also see that thermal entanglement especially between NNN qubits can be established and sustained over a wide range of temperature T by frustration parameter α.

This work is organized as follows: In Section 2, the Hamiltonian of the system is introduced and then concurrence is given. In Section 3, we discuss the ground state entanglement between NN and NNN qubits. In Section 4, thermal entanglement between NN and NNN qubits is computed and analyzed as a function of the control parameters. In Section 5, our concluding remarks are summarized.

2. The model

A four qubit anisotropic J_1–J_2 Heisenberg XXZ spin system with DM interaction is described by the Hamiltonian

$$H = \frac{J_1}{2}\sum_{i=1}^{4}(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta_1\sigma_i^z\sigma_{i+1}^z) + \frac{J_2}{2}\sum_{i=1}^{4}(\sigma_i^x\sigma_{i+2}^x + \sigma_i^y\sigma_{i+2}^y + \Delta_2\sigma_i^z\sigma_{i+2}^z) + \frac{D}{2}\sum_{i=1}^{4}(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x), \tag{2}$$

where $\sigma_i^\kappa (\kappa = x, y, z)$ are the Pauli matrices of the ith qubit and the periodic boundary conditions $\sigma_{i+1} = \sigma_1$ and $\sigma_{i+2} = \sigma_2$ are adopted. $J_1 = J$ and $J_2 = J\alpha$ are the coupling constants ($J_1, J_2 > 0$ correspond to antiferromagnetic (AFM) case, $J_1, J_2 < 0$ correspond to ferromagnetic (FM) case) between the nearest neighboring (NN) and the next nearest neighboring (NNN) qubits, respectively. $\alpha = J_2/J_1$ is termed as the frustration parameter. Δ_1 and Δ_2 represent the anisotropy parameters in the z-direction between NN and NNN qubits, respectively. Note furthermore that D is the z-component of the Dzialoshinskii–Moriya (DM) anisotropic and antisymmetric exchange interaction [40,41].

In the standard basis $\{|0000\rangle, |0001\rangle, |0010\rangle, |0011\rangle, |0100\rangle, |0101\rangle, |0110\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1010\rangle, |1011\rangle, |1100\rangle, |1101\rangle, |1110\rangle, |1111\rangle\}$, the eigenvalues of H are obtained as

$$E_{1,2} = 2(D - J\alpha), \tag{3}$$

$$E_{3,4} = -2(D + J\alpha), \tag{4}$$

$$E_{5,6} = 2J(-1 + \alpha), \tag{5}$$

$$E_{7,8} = 2J(1 + \alpha), \tag{6}$$

$$E_{9,10} = -2J\alpha\Delta_2, \tag{7}$$