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# Gauge invariant lattice quantum field theory: Implications for statistical properties of high frequency financial markets

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ABSTRACT

We report on initial studies of a quantum field theory defined on a lattice with multi-ladder geometry and the dilation group as a local gauge symmetry. The model is relevant in the cross-disciplinary area of econophysics. A corresponding proposal by Ilinski aimed at gauge modeling in non-equilibrium pricing is implemented in a numerical simulation. We arrive at a probability distribution of relative gains which matches the high frequency historical data of the NASDAQ stock exchange index.

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## 1. Introduction

The concept of a gauge theory revolves around the notion that the laws of physics must be independent of arbitrary choices made by an observer, such as the scales (units) of length and time, or the choice of some coordinates in general.

Gauge invariance came into prominence in 1929 through the work of Weyl [1] in a quest to understand gravitation. Since then, it has evolved beyond its original context to also become a cornerstone of modern quantum field theory. The gauge principle is realized in terms of a local symmetry group $G$ which reflects some aspect of the physics. Famously, quantum electrodynamics (QED) is locally gauge invariant with respect to $G = U(1)$, meaning that the choice of the complex phase for matter fields is arbitrary. Indeed, our current understanding of particle physics as a unified theory of electroweak and strong interactions, the so-called standard model, is built on a gauge principle combined with a specific pattern for (spontaneous) symmetry breaking.

Methods of theoretical physics have found their way into financial mathematics and related fields of economics [2]. The term "econophysics" has been coined to describe this area of research [3]. A typical application is concerned with stochastic properties of historical (market) data. This explains the connection to quantum physics, which is based on a stochastic interpretation of physical phenomena. The marriage of financial mathematics with quantum physics has been called quantum finance [4]. Quantization is best achieved using path integrals [5] because this guarantees viability even in nonperturbative systems, like off-equilibrium markets. Moreover, numerical simulation on some lattice geometry is readily implemented.

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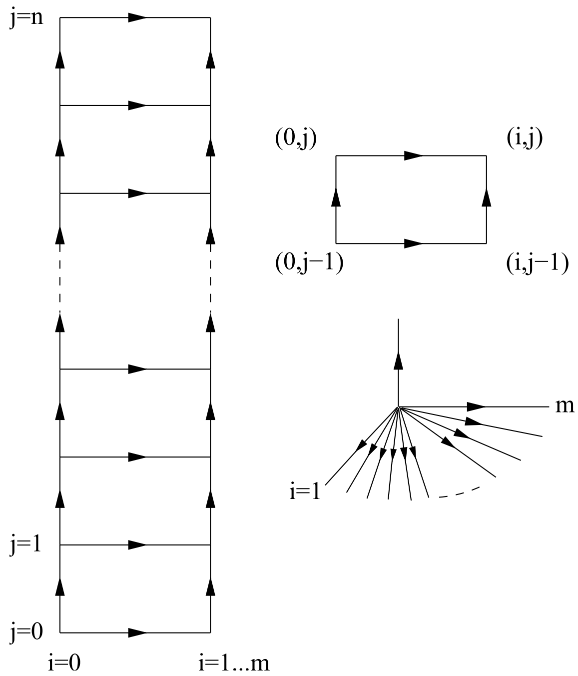


**Fig. 1.** Illustration of the geometry of the lattice model and the label scheme for the sites.

The idea of combining the gauge principle with quantum field theory in the context of econophysics has been advanced by Ilinski [6]. Its rationale is that traders in financial markets, for example, interact in similar ways with the trading environment, irrespective of the units used for the traded commodity. For example, market observables like the distribution of relative gains to an investor should be similar regardless of whether the currency unit used in the exchange is USD, Euro, Yen, or some other unit, provided that an appropriate scale factor has been applied. Empirical evidence related to this idea comes from the observation of scaling of the market data with respect to different time horizons [7], yet another scale.

A somewhat independent path of using the concept of a gauge in stochastic investment models in mathematical finance has been pursued by Smith and Speed [8] and Farinelli [9]. The concepts of differential geometry and stochastic mathematics are combined in a rather formal way to yield interesting theoretical results. Comparing our and Ilinski's [6] approaches requires a translation of terminology: The terms "deflator and term structure" refer to "gauge fields", though in principle the concepts are similar. We prefer Ilinski's approach over Farinelli's, in part because it more directly lends itself to practical application.

The present paper is concerned with an implementation of those ideas in the framework of a numerical simulation of a simple gauge invariant lattice quantum field theory. Subsequently, we will describe the lattice model, its interpretation in the context of financial markets, and the numerical implementation, and then proceed to matching the results to historical data of the NASDAQ stock index at one-minute intervals.

## 2. The lattice model

Inspired by Ref. [6] we work with a lattice geometry as illustrated in Fig. 1. In a three-dimensional rendering, the vertical direction represents time with slices $j = 0, 1, \ldots, n$ separated by some arbitrary unit. At each time slice, we imagine a horizontal plane in which there are space locations. A second index $i = 0$ labels the location of the origin and $i = 1, \ldots, m$ indicates (different) locations one step away from it. It is useful to divide the lattice sites into two sets

$$\mathcal{X}_0 = \{(0, j) | j = 0, \ldots, n\} \tag{1}$$

$$\mathcal{X}_1 = \{(i, j) | i = 1, \ldots, m, j = 0, \ldots, n\}. \tag{2}$$

The sites are connected through three sets of links. In the space direction (horizontal) we have

$$\mathcal{L}^s = \{(0, j) \to (i, j) | i = 1 \ldots m, j = 0, \ldots, n\}, \tag{3}$$

and in the time direction (vertical), we distinguish two sets of links

$$\mathcal{L}_0^t = \{(0, j-1) \to (0, j) | j = 1, \ldots, n\} \tag{4}$$

$$\mathcal{L}_1^t = \{(i, j-1) \to (i, j) | i = 1, \ldots, m, j = 1, \ldots, n\}. \tag{5}$$