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Uncertainty of cooperation in random scale-free networks

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ABSTRACT

We study the spatial prisoner's dilemma game where the players are located on the nodes of a random scale-free network. The prisoner's dilemma game is a powerful tool and has been used for the study of mutual trust and cooperation among individuals in structured populations. We vary the structure of the network and the payoff values for the game, and show that the specific conditions can greatly influence the outcome of the game. A variety of behaviors are reproduced and the percentage of cooperating agents fluctuates significantly, even in the absence of irrational behavior. For example, the steady state of the game may be a configuration where either cooperators or defectors dominate, while in many cases the solution fluctuates between these two limiting behaviors.

1. Introduction

Game theory has found many exciting applications in fields as diverse as biology, physics, economics and social systems, in describing the complex dynamics of these systems [1]. Of particular importance is the pursuance of the understanding the emerging cooperation (or defection) in the social behavior of individuals. The prisoner's dilemma (PD) game, despite its simplicity, is among the most widely used archetypal models. PD has been studied in numerous variations and many different strategies have been proposed for achieving maximum benefit for the participants.

In its basic version, PD is a game between two contestants trying to maximize their profit [2]. Nowak and May introduced the spatial PD game, where now many individuals play the game simultaneously [3]. The players interact with their 'neighbors', i.e. the individuals that are closer to a given player when all participants are placed on the vertices of a lattice. This version of the game provided a new insight and important collective phenomena were observed. Although the defection state is in general stable, the spatial game provided the conditions for a stable cooperation state, as well. The spatial game was extended in the case where the substrate for the players was a small-world network, and players would interact with individuals that could be located far from each other [4]. Holme et al. also studied the influence of a scale-free network substrate on the game by using empirical data for some real-world social networks [5]. A large number of publications also studied variations of the game played on a scale-free substrate [6–10].

In the present study the original version of the spatial PD game with imitation dynamics is studied using a network of contacts that is scale-free [11]. A number of different methods can be used for a node to decide what strategy should be followed next, depending on the circumstances that are simulated. Here, the total payoff of a node during one round is used for determining the most successful strategy without averaging over the number of the node contacts. In this scheme the hub strategy is expected to dominate. For example, it is a well-documented feature in a society that many people tend to follow some of the most exposed members of this society, such as teenagers who readily adopt the lifestyle of successful actors, rock-stars etc. In contrast, the most influential nodes are much less likely to reverse their strategy because they already earn a large profit via their interaction with many neighbors. Only when a sufficiently large number of neighbors use a different

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strategy than the hub at a given time will the hub change its strategy. In the real world this translates to popular members of the society changing their attitude if most of their followers do so before them, for fear of being eventually isolated.

In Ref. [5] the authors studied a similar problem, using three different real-world acquaintance networks as the substrate for the interaction between the players. They also used a probability of irrational decision p_m, where a player chooses the opposite strategy from the one adopted by the most successful player during the previous step, in order to drive the system. Here we show that the generic model of imitation dynamics on a scale-free network, even in the absence of irrational behavior, leads to a level of cooperation unknown in advance with wide and usually bimodal distributions, with the result that cooperation may equally be either the prevailing or discouraged attitude on the network. Hubs play an important role in this, but the dominant attitude may also emerge via the cooperation of many lower degree nodes.

2. The model

The agents playing the game are placed on the nodes of a scale-free network. The connectivity of the network nodes obeys a power law of the form $P(k) \sim k^{-\gamma}$, where γ is a parameter with typical values in the range $\gamma = 2$–4. For the creation of a network we use a configuration model where no self-links or double connections between a given pair of nodes are allowed. The network comprises N nodes and it is static, meaning that no links are subsequently created or destroyed. Only the largest cluster of the network is used, which corresponds to 35%–100% of the system nodes, depending on the value of γ.

Initially, each player is assigned a strategy, which can be either cooperation (C) or defection (D). During one round of the game each individual plays the PD game with all neighbors. The payoff for each player is determined by the strategy that they and their interacting neighbor follow.

The status of a node i during the time step t can be described by a vector x_i which can assume one of two values:

$$x_i^C(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad x_i^D(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1}$$

indicating cooperation or defection, respectively. For the payoff matrix we use the simplified form

$$A = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}. \tag{2}$$

The total gain for a node i during the time step t is then calculated via

$$g_i(t) = \sum_j x_i^{\mathrm{T}}(t) A x_j(t), \tag{3}$$

where j runs over all the node neighbors, i.e. nodes that are directly connected to i via a single link (excluding i itself).

The above means that if both players choose to cooperate they both earn a reward of 1. If they both defect, they are both punished and earn 0. If one chooses to defect while the other cooperates, the defector receives the temptation earning b (where $b > 1$), while the cooperator is left with the sucker payoff, i.e. 0. It is important to notice that an individual cares about maximizing their own payoff, rather than simply gaining more than their opponents, in which case they would always defect.

All agents follow an imitation behavior. Each player calculates their total payoff during one round and then inspects the corresponding payoff of all their neighbors. This player then adopts the strategy of their most successful neighbor (i.e. the one with the highest payoff), which may well be their own strategy. One time step consists of one network sweep, where all players interact with their neighbors and decide on their strategy during the next round.

The order parameter in our system is the percentage of players $\rho_C(t)$ that choose to cooperate at a given time t:

$$\rho_C(t) = \frac{1}{N} \sum_{i=1}^{N} (1\,0) \cdot x_i(t). \tag{4}$$

For a given network realization and given initial conditions we perform 10^5 rounds of the play. The first 9×10^4 rounds are used for relaxation and for the results we use the remaining 10^4 values of $\rho_C(t)$. We then average over 10^4 different network realizations of a network for a given γ value and repeat the entire process for other γ values, as well.

3. Results

It is known [3] that for a given temptation value b the spatial PD game on a two-dimensional lattice always converges to a given asymptotic ρ_C value that only depends on the exact rules of the game, such as the number of interacting neighbors. The initial distribution of cooperators and defectors in the lattice does not influence the result, even in the extreme case of a single defector emerging in a lattice full of cooperators. For example, considering an interaction with eight nearest neighbors yields a value of $\rho_C = 1$ in the range $1 < b < 1.8$ and $\rho_C = 0.318$ when $1.8 < b < 2$. On scale-free networks it has been shown, and our simulations confirm this, that the behavior is entirely different. The percentage of cooperators