Physica A 419 (2015) 659–667

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Optimising threshold levels for information transmission in binary threshold networks: Independent multiplicative noise on each threshold

Bingchang Zhou [a,*], Mark D. McDonnell [b]

[a] Department of Applied Mathematics, Northwestern Polytechnical University, Xian 710072, PR China

[b] Computational and Theoretical Neuroscience Laboratory, Institute for Telecommunications Research, University of South Australia, SA 5095, Australia

HIGHLIGHTS

- We have found the optimal threshold levels in a multilevel binary threshold system with multiplicative and additive noise.
- This mode is relevant to sensor network design and understanding neurobiological sensory neurons such as in the peripheral auditory system.
- The optimal thresholds for multiplicative noise exhibit bifurcations.
- The optimal thresholds are all unique when the additive noise intensity is fixed.
- The optimal thresholds are identical for large additive noise.

ARTICLE INFO

Article history:
Received 14 May 2014
Received in revised form 11 October 2014
Available online 24 October 2014

Keywords:
Suprathreshold stochastic resonance
Stochastic pooling network
Noise
Multilevel threshold system
Bifurcation
Mutual information

ABSTRACT

The problem of optimising the threshold levels in multilevel threshold system subject to multiplicative Gaussian and uniform noise is considered. Similar to previous results for additive noise, we find a bifurcation phenomenon in the optimal threshold values, as the noise intensity changes. This occurs when the number of threshold units is greater than one. We also study the optimal thresholds for combined additive and multiplicative Gaussian noise, and find that all threshold levels need to be identical to optimise the system when the additive noise intensity is a constant. However, this identical value is not equal to the signal mean, unlike the case of additive noise. When the multiplicative noise intensity is instead held constant, the optimal threshold levels are not all identical for small additive noise intensity but are all equal to zero for large additive noise intensity. The model and our results are potentially relevant for sensor network design and understanding neurobiological sensory neurons such as in the peripheral auditory system.

1. Introduction

Stochastic resonance (SR) occurs when the presence of noise in a nonlinear system provides improved signal transmission and/or detection [1,2]. Since first proposed by [3,4], SR has been studied in many nonlinear systems in fields ranging from physics and chemistry to neuroscience and biophysics, for a wide variety of different noise types [5–10]. Different

* Corresponding author.
E-mail address: bczhou98@126.com (B. Zhou).

http://dx.doi.org/10.1016/j.physa.2014.10.074
0378-4371/

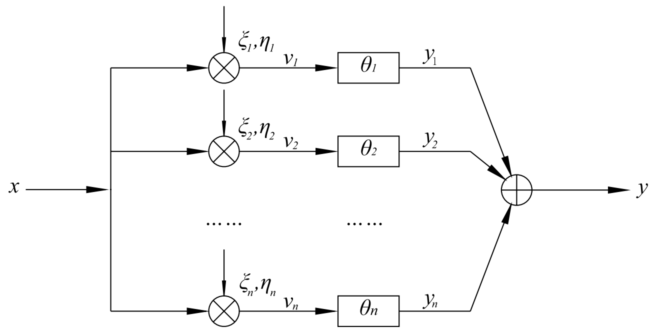

Fig. 1. System of N threshold devices that each operate on a common input random signal, x, subject to *i.i.d.* multiplicative noise, ξ_i, and *i.i.d.* additive noise, η_i. The overall output, y, is the sum of the N binary responses, y_i. We seek to optimise the threshold values, $\theta_i\ i = 1, \ldots, N$ with respect to the mutual information between x and y.

varieties of SR exist, such as quantum stochastic resonance [11], doubly stochastic resonance [12], non-Markovian stochastic resonance [13] and aperiodic stochastic resonance [14].

In 2000, a new form of SR called suprathreshold stochastic resonance (SSR) was introduced [15]. This occurs in a parallel array of threshold devices, whose outputs are summed [16]. In Ref. [15], each threshold operates on the same Gaussian signal, but is subject to independent and identically distributed (*i.i.d.*) additive Gaussian noise, and the SSR effect is measured using mutual information. Since this seminal work, SSR has been extensively investigated for the case where all thresholds have the same value [17–27]. Closely related work has focused on the interesting case when threshold devices are replaced by identical neuron models [28–33]. However, other work has relaxed the constraint of identical thresholds [34], and found the optimal thresholds for maximising the mutual information as a function of additive noise intensity. Subsequently, the parallel threshold array setup was generalised into the concept of *stochastic pooling networks* [35].

In 2007, Nikitin et al. [26] studied SSR in an array of threshold devices subject to *i.i.d. multiplicative* Gaussian noise. They found that the information transmission of weak signals is significantly better with multiplicative noise and gave a robust method of transmitting information.

In this paper, we extend this body of work by considering independent multiplicative noise in each threshold device, and aiming to optimise the threshold levels. We consider both the case of multiplicative noise alone, as well as the combination of additive and multiplicative noise. The structure of this paper is as follows. Section 2 describes the model and problem definition, and provides a brief summary of previous results for optimisation of the threshold system subject to additive noise. We then in Section 3 present results for the optimal thresholds when the system is subject to only multiplicative noise. We consider two cases: Gaussian noise and uniform noise. Next, Section 4 presents results for the optimal thresholds of the system when there is both additive and multiplicative Gaussian noise. Section 5 provides discussion and conclusions.

2. Model and optimisation problem

A schematic model of the threshold system is shown in Fig. 1. This system consists of N simple threshold units that provide binary outputs, $y_i \in \{0, 1\}$. Each unit receives a signal composed from *i.i.d.* samples, x, from the probability density function $f_x(x)$. The ith threshold unit is subject to *i.i.d.* multiplicative noise and/or *i.i.d.* additive noise. The mutual information between the input signal, x and output signal, y, of the N-level threshold system can be written as in Ref. [15]:

$$I(x, y) = H(y) - H(y|x)$$
$$= -\sum_{n=0}^{N} P_y(n) \log_2 P_y(n) - \int_{-\infty}^{\infty} f_x(x) \sum_{n=0}^{N} P_{y|x}(n|x) \log_2 P_{y|x}(n|x), \quad (1)$$

where $H(y)$ is the entropy of the output signal, $H(y|x)$ is the average conditional output entropy, $P_{y|x}(n|x)$ is the conditional probability mass function of output y given input x. Finally, $P_y(n)$ is the output probability mass function, which can be expressed as

$$P_y(n) = \int_{-\infty}^{\infty} P_{y|x}(n|x) f_x(x)\, dx, \quad n = 0, \ldots, N. \quad (2)$$

The problem we aim to solve is to find the vector of N thresholds, $\boldsymbol{\theta}$, that maximises the mutual information in the multi-threshold system. This can be expressed as the following optimisation problem:

$$\text{Find:} \quad \max_{\boldsymbol{\theta}} I(x, y)$$
$$\text{subject to:} \quad \boldsymbol{\theta} \in R^N. \quad (3)$$