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Effects of Gaussian colored noise on time evolution of information entropy in a damped harmonic oscillator

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HIGHLIGHTS

- We studied the effects of Gaussian colored noise on a damped harmonic oscillator.
- One-dimensional non-Markovian process is stochastically equivalent to two-dimensional Markovian process.
- The dimension of Fokker–Planck equation is reduced by the linear transformation.
- The exact expression of the time dependence of information entropy is derived.
- The present calculation can be used to interpret the interplay of noise on information entropy.

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ABSTRACT

The effects of Gaussian colored noise on time evolution of information entropy in a damped harmonic oscillator are studied in this paper. The one-dimensional non-Markovian process with Gaussian colored noise is stochastically equivalent to two-dimensional Markovian process and the dimension of Fokker–Planck equation is reduced by the linear transformation. The exact expression of the time dependence of information entropy is derived on the basis of Fokker–Planck equation and the definition of Shannon's information entropy. The relationship between the properties of damping constant, the frequency of the oscillator and Gaussian colored noise and their effect on time evolution of information entropy is also discussed.

1. Introduction

Damped harmonic oscillators have attracted noticeable attention due to their wide application in many fields, such as quantum optics, quantum field theory, solid theory, and quantum mesoscopic circuit theory [1,2]. The internal dynamics of a force-free harmonic oscillator with additive or multiplicative noise was widely introduced as a model to understand the different phenomena in physics, biology, economics, and so on [1–7]. However, in most of the models discussed in the literature, the dynamical equations include a deterministic driving force. It is well known that the nature of random force (e.g. noise) may influence the dynamical systems in many aspects, such as stationary probability [7–9], escape rate [9,10], noise-induced phase transitions [11,12], stochastic resonance [4,5,13–15] and the time derivative of information entropy [6,16–23]. The specific nature of the stochastic process may play an important role in the process of equilibration for a given non-equilibrium state of the noise-driven dynamical system. Thus an understanding of the interplay among frictional force, random force, noise strength and external force, if any, has become a subject in the recent past.

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Entropy is an appropriate tool in the study of the nature of dynamical systems driven by noise, and one can use the entropy as the witness of non-equilibrium state. The concept of entropy is introduced in accordance with the quantitative description for the natural evolution of the quasi-static isolated system and it plays an important role in describing the evolution, instability, disorder or confusion and the information transmission of stochastic systems [23–26]. In recent years, studying the information entropy and its related quantities has become an important issue for the understanding of noise-driven dynamical systems in detail, such as Refs. [6,16–23]. The time evolution of information entropy mainly considers the signature of the rate of phase space expansion and contraction of the stochastic process. This implies that the specific nature of the random process has a strong role to play with information entropy. In view of the immediate connection between information entropy and probability distribution function of the phase space variables, it is worthwhile to inquire about the imprints of the nature of noise on entropy [16–18].

The objective of the present paper is to show the effects of Gaussian colored noise on time evolution of information entropy in a damped harmonic oscillator. This paper is organized as follows: In Section 2, the damped harmonic oscillator driven by Gaussian colored noise is introduced and the dimension of Fokker–Planck equation is reduced by the way of linear transformation. In Section 3 the exact expression of the time dependence of information entropy is obtained based on the definition of Shannon's information entropy. In Section 4, the relationship between the properties of damping constant, the frequency of the oscillator and Gaussian colored noise and their effect on information entropy are discussed. And some conclusions are drawn in Section 5.

2. The Fokker–Planck equation of a damped harmonic oscillator driven by Gaussian colored noise

The relevant Langevin equation of a damped harmonic oscillator driven by Gaussian colored noise is as follows [1–6]:

$$\ddot{x} = -\gamma\dot{x} - \omega^2 x + \eta(t), \quad (\gamma > 0), \tag{1}$$

where, γ $(\gamma > 0)$ is the damping constant, ω is the frequency of the oscillator. $\eta(t)$ is Gaussian colored noise and

$$\langle \eta(t)\,\eta(s)\rangle = \frac{D}{\tau}\exp\left(\frac{|t-s|}{\tau}\right). \tag{2}$$

Here D and τ denote the intensity and the correlation time of the Gaussian colored noise $\eta(t)$. When $D = \gamma kT$ (k is Boltzmann's constant and T is the temperature), the system by Eqs. (1) and (2) is sometimes termed as thermodynamically closed system.

The one-dimensional non-Markovian process (1) with Eq. (2) is stochastically equivalent to two-dimensional Markovian process

$$\begin{aligned} \ddot{x} &= -\gamma\dot{x} - \omega^2 x + \eta(t), \\ \dot{\eta} &= -\frac{1}{\tau}\eta + \frac{1}{\tau}\xi(t), \end{aligned} \tag{3}$$

where $\xi(t)$ is Gaussian white noise with

$$\langle \xi(t)\rangle = 0, \qquad \langle \xi(t)\,\xi(s)\rangle = 2D\delta(t-s). \tag{4}$$

Let $X_1 = x$, $X_2 = \dot{X}_1$ and $X_3 = \eta$, Eq. (3) can be rewritten as

$$\begin{aligned} \dot{X}_1 &= X_2, \\ \dot{X}_2 &= -\omega^2 X_1 - \gamma X_2 + X_3, \\ \dot{X}_3 &= -\frac{1}{\tau}X_3 + \frac{1}{\tau}\xi(t). \end{aligned} \tag{5}$$

The Fokker–Planck equation of Eq. (5) can be written as

$$\frac{\partial\rho(X_1,X_2,X_3,t)}{\partial t} = -\frac{\partial X_2\rho}{\partial X_1} + \frac{\partial}{\partial X_2}\left(\omega^2 X_1 + \gamma X_2 - X_3\right)\rho + \frac{1}{\tau}\frac{\partial X_3\rho}{\partial X_3} + \frac{D}{\tau^2}\frac{\partial^2\rho}{\partial X_3^2} \tag{6}$$

where, $\rho(X_1,X_2,X_3,t)$ is the extended phase-space probability distribution function.

Making use of the linear transformation $U = aX_1 + bX_2 + X_3$ [6,16–23,27], Eq. (6) can be rewritten as

$$\frac{\partial\rho(U,t)}{\partial t} = -\frac{\partial A\rho}{\partial U} + B\frac{\partial^2\rho}{\partial U^2}. \tag{7}$$

Here

$$A = -\lambda U, \qquad B = \frac{D}{\tau^2}, \tag{8}$$

$$\lambda U = b\omega^2 X_1 + (b\gamma - a)X_2 + \left(\frac{1}{\tau} - b\right)X_3 \tag{9}$$