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A new perspective to formulate a dissipative thermo field dynamics

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HIGHLIGHTS

- Thermal dissipations are studied from the view point of thermo field dynamics.
- Thermal dissipations are renormalized into the non-Hermitian interactions between the original and tilde spaces.
- We propose a new perspective of the double Hilbert space on the thermo field dynamics as a heat bath.
- The present new perspective is applied to a many-body system in order to clarify thermal fluctuations.
- In general, the tilde space does not correspond to the heat bath directly, but corresponds to it through effective non-Hermitian interactions.

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ABSTRACT

In the present study, we propose a new perspective on thermal dissipation based on the thermo field dynamics. From the view point of the renormalization theory, there appear effective interactions between the original and tilde spaces on thermo field dynamics with reducing thermal disturbances. This study yields the equivalence of the following two pictures, namely such a spin system with a random field due to a heat bath as is defined in a Hilbert space and a finite-size system with effective interactions defined in a double Hilbert space. The correspondence of the above two systems yields such perspective that the thermal disturbance is described by the effective non-Hermitian interactions.

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1. Introduction

Recently, thermo field dynamics (TFD) [1–4] played a very useful and important role to study entanglements [5–11], Kondo effects [12], the entropy effect of a black hole [13,14] and non-equilibrium phenomena [5,6,15–17]. We have to introduce a tilde space in order to study statistical properties using TFD. Actually, the state vector $|\Psi\rangle$, namely "TFD state vector", is defined [1–4] as

$$|\Psi\rangle = \rho^{1/2}|I\rangle \quad \text{and} \quad |I\rangle = \sum_n |n\rangle \otimes |\tilde{n}\rangle \equiv \sum_n |n, \tilde{n}\rangle \tag{1}$$

with the density matrix ρ of the relevant system. The bases $|n\rangle$ (or $|\tilde{n}\rangle$) are the eigenstates of the Hamiltonian (or $\tilde{H}$ in the tilde space) with the eigenvalues $\{E_n\}$, namely $\mathcal{H}|n\rangle = E_n|n\rangle$ (or $\tilde{\mathcal{H}}|\tilde{n}\rangle = E_n|\tilde{n}\rangle$). One of the present authors [18,19] proved

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that the state $|I\rangle$ itself is invariant for any representation, namely

$$|I\rangle = \sum_{\alpha} |\alpha\rangle \otimes |\tilde{\alpha}\rangle \equiv \sum_{\alpha} |\alpha, \tilde{\alpha}\rangle \tag{2}$$

where $\{\alpha\}$ is an arbitrary orthogonal complete set. Then we can easily derive [4,18,19] the thermal average as

$$\langle \Psi | Q | \Psi \rangle = \sum_{\alpha,\alpha'} \langle \alpha, \tilde{\alpha} | \rho^{1/2} Q \rho^{1/2} | \alpha', \tilde{\alpha}' \rangle = \mathrm{Tr} \rho Q = \langle Q \rangle \tag{3}$$

for the physical quantity Q. Especially, once the tilde vector $|\tilde{\alpha}\rangle$ satisfies the relation $|\tilde{\alpha}'\rangle = \sum_{\alpha} U^{*}_{\alpha',\alpha} |\tilde{\alpha}\rangle$ for $|\alpha'\rangle = \sum_{\alpha} U_{\alpha',\alpha} |\alpha\rangle$, one can find clearly the "general representation theorem" [4,18,19]. The general representation theorem means not only the thermal average $\langle Q \rangle$ but also the representations of TFD state vectors are independent of the bases $\{|\alpha\rangle\}$, because [4,18,19]

$$|I'\rangle = \sum_{\alpha'} |\alpha', \tilde{\alpha}'\rangle = \sum_{\alpha'} \sum_{\alpha_1,\alpha_2} U^{*}_{\alpha',\alpha_1} U_{\alpha',\alpha_2} |\alpha_1, \tilde{\alpha}_2\rangle = \sum_{\alpha_1,\alpha_2} \delta_{\alpha_1,\alpha_2} |\alpha_1, \tilde{\alpha}_2\rangle = |I\rangle. \tag{4}$$

Furthermore, this general representation theorem derives [4,18,19] the time development equation

$$\mathrm{i}\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{\mathcal{H}} |\Psi(t)\rangle \tag{5}$$

of the TFD state vector $|\Psi(t)\rangle$ using the von Neumann equation

$$\mathrm{i}\hbar \frac{\partial}{\partial t} \rho(t) = [\mathcal{H}, \rho(t)]. \tag{6}$$

This is one of the important merits of the general representation theorem [4,18,19]. Here $\hat{\mathcal{H}}$ is defined as $\hat{\mathcal{H}} = \mathcal{H} - \tilde{\mathcal{H}}$. Additionally, we have introduced [6] the extended density matrix $\hat{\rho} \equiv |\Psi\rangle \langle\Psi|$ and found a simple way to study the entanglement directly even in a dissipative system. On the above discussions of TFD, a Hilbert space defined by a set of bases $\{|\alpha\rangle\}$ (namely "original space") is extended to a double Hilbert space which is defined by a direct product space $\{|\alpha\rangle \otimes |\tilde{\alpha}\rangle\}$ [4]. Note that the tilde space $\{|\tilde{\alpha}\rangle\}$ needs to be isomorphic to the original space $\{|\alpha\rangle\}$.

In this way, the tilde space is introduced in a mathematical point of view, and then its physical meanings are not so clear. Nonetheless, the double Hilbert space is very useful practically. For example, it was applied to the density matrix renormalization group (DMRG) analysis and multi-scale entanglement renormalization ansatz (MERA) [8,9] at finite temperatures [7,10,11], and it was used to study Kondo effect [12] and to study a black hole entropy [13,14]. There are some common features in these previous studies, in which the thermal effect can be treated by introducing the tilde space (or double Hilbert space). In the present study, we try here to clarify how the thermal noises are renormalized into TFD pictures. This study does not clarify the origin of thermal disturbances from the viewpoint of first principles but clarifies phenomenologically the renormalization scheme of thermal noises using TFD.

In the following section, by treating a simple example, we obtain an effective interaction between the original and tilde spaces. It may be trivial that effective interactions appear owing to renormalization procedure. However, these effective interactions enable us to understand a plausible treatment of a heat bath and they support the previous suggestions [15,20–22] for the dissipative formulation of TFD. In Section 3, the present new perspective is applied to many-body systems. Summary and discussions are included in Section 4.

2. Observation of new perspective on a simple example with a thermal noise

In this section, we present a new perspective of tilde space using a simple example. In 1961, Schwinger [23] showed that effects of a heat bath can be represented by a thermal noise, using a fluctuation propagator of quantum oscillators. Thus, we adopt a single spin system with the thermal noise $R(t)$ described by the following Hamiltonian including a spin variable S_0;

$$\mathcal{H} = \mathcal{H}_0 - JR(t)S_0, \tag{7}$$

where the thermal noise $R(t)$ is regarded as the white Gaussian noise, namely

$$\langle R(t)\rangle_R = 0 \quad \text{and} \quad \langle R(t)R(t')\rangle_R = \epsilon\delta(t - t'). \tag{8}$$

$\mathcal{H}_0$ is a Hamiltonian of interest without noise. Here the notation $\langle A\rangle_R$ denotes the random average of the stochastic parameter A, and the noise intensity is denoted by the parameter ϵ. The constant real number J denotes a kind of coupling constant. For the Hamiltonian (7), the tilde Hamiltonian $\tilde{\mathcal{H}}$ [18] is obtained by

$$\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_0 - JR(t)\tilde{S}_0. \tag{9}$$

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