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Analyzing the financial crisis using the entropy density function

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HIGHLIGHTS

- We analyze the uncertainty in financial markets.
- We analyze the patterns of extreme cases.
- We find that the entropy can capture the financial crisis.

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ABSTRACT

The risk that is created by nonlinear interactions among subjects in economic systems is assumed to increase during an abnormal state of a financial market. Nevertheless, investigating the systemic risk in financial markets following the global financial crisis is not sufficient. In this paper, we analyze the entropy density function in the return time series for several financial markets, such as the S&P500, KOSPI, and DAX indices, from October 2002 to December 2011 and analyze the variability in the entropy value over time. We find that the entropy density function of the S&P500 index during the subprime crisis exhibits a significant decrease compared to that in other periods, whereas the other markets, such as those in Germany and Korea, exhibit no significant decrease during the market crisis. These findings demonstrate that the S&P500 index generated a regular pattern in the return time series during the financial crisis.

1. Introduction

Following the US subprime crisis, understanding the stability of financial systems has become a significant research area in the fields of economics, physics and mathematics [1–26]. In particular, the abnormal behaviors in economic systems have been analyzed using the concepts and methods developed from complex systems and econophysics. Based on the previous studies of the financial crisis as a complex dynamical system, there are many examples in the literature of sharp transitions from stable states to very different states. Although the events created in the financial industry sectors were the cause of the global financial crisis, we need to determine potentially relevant factors to reduce the negative effects to economic systems. However, only recently attention has been focused on the interactions among subjects in financial systems. In this paper, we investigate the specific issue of how to measure the stability of economic systems. For this purpose, we explore the relationship between the distribution function of patterns in the return time series of international indices, such as

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the S&P500, KOSPI and DAX stock markets, and the financial crisis, where pattern is defined as weekly consecutive binary variables: $+1$ for the positive log return value and 0 for the negative log return value in return time series. Generally, the risk in the financial system is defined by the uncertainty of a return and amount of financial loss. In the economic system, we need to know the systematic risk which cannot erase whole risk of financial product to determine standard price. Therefore, it is important that we measure the risk using a valid method. The volatility as a market risk is widely accepted in financial field and denoted as highly unpredictable. The standard deviation of the return time series is able to measure the market risk based on a situation that is extremely random state in the stock price movement. In this case, the entropy value as the risk measure is proportional to the volatility [27], whereas these two measures are quite different if the stock price cannot be described by the random walk process. The argument, non-random walk hypothesis, is supported by numerous studies in the econophysics field. Here, we establish the entropy method for quantifying the degree of stability of a financial market. The relevant measurement should generally reveal the actual causes responsible for the recent financial crisis and what will prevent a possible financial crisis in the future.

More recently, the correlations among the global market indices have been widely used to measure the state of markets [28]. W. Chen et al. [29] proposed a new approach to the multifractal volatility method to quantify the contagion effect of the US subprime crisis on the global economic systems, showing that the Chinese stock market was significantly influenced by the subprime mortgage crisis. The Lévy processes have been applied for extracting useful information regarding the underlying assets from the S&P500 index options, revealing that the risk preferences of equity investors were indicating a market abnormality before the US subprime crisis, anticipating the downfall of the equity market in 2008 and the recovery to normal levels in 2009 [30].

This paper contributes to the literature as follows. First, we rely on the statistical properties of various patterns in the return time series to measure the degree of stability in the financial market. We find that the probability density function (PDF) of each pattern generated with length $L = 6$ in the stock markets coincides with the subprime crisis between 2007 and 2008, where the unit of length is days. In this paper, we use $L = 6$ in order to observe the weekly up down patterns in log returns. Even if we use $L = 4$, 5, or 6, there is no significant difference of the probability distribution of patterns. The reason for using $L = 6$ is that with a small L the resolution of probability distribution of patterns becomes poor. Second, we analyze the entropy using the PDF of all possible patterns and mark a benchmark in the subprime crisis. Third, we find that the risk estimated by the entropy in the S&P500 index exhibits a meaningful decrease in the subprime crisis compared to normal market states, whereas there is no significant diminish in the DAX and KOSPI markets.

This paper is organized as follows: Section 2 briefly describes the entropy methodology and data used. Section 3 presents the empirical results of the entropy of the equity markets. Finally, Section 4 concludes the paper.

2. Data and methodology

We investigate market uncertainty and risks in the financial market using the daily data of the markets from Oct. 1, 2002, to Dec. 31, 2011, for indices such as the S&P500, KOSPI, and DAX (Fig. 1). The log return is defined as

$$r(t) = \log p(t + \Delta t) - \log p(t), \tag{1}$$

where $p(t)$ is the price at time t and Δt is the time lag. To analyze the risk in the financial market, we observe the volatility of the log return of three different stock markets (Fig. 3), which is defined by

$$\sigma_r^2 = \left\langle r\,(t)^2\right\rangle - \langle r\,(t)\rangle^2\,, \tag{2}$$

where the fluctuations in volatility are calculated from the past time series of the log return for a period of one year (Fig. 2).

We measure the degree of randomness and the uncertainty of the financial time series using the Shannon entropy.

Shannon [23] first suggested that entropy is a measure of the uncertainty associated with a random variable in information theory, and entropy has since been widely used to measure the degree of randomness and uncertainty of financial time series data. The Shannon entropy of the discrete random variable X with a probability function $P(X)$ can be written as

$$H(X_L) = -\sum_i P\left(X_L^i\right) \log_2 P\left(X_L^i\right), \tag{3}$$

where X is a variable of L consecutive random variables $X_L^i = (s_i, s_{i+1}, \dots, s_{i+L-1})$. We denote the variable s_i as taking two values: $+1$ for the positive log return value and 0 for the negative log return value on the ith day in the financial time series. We use the normalized entropy density $h(L)$ because $H(X_L)$ increases as L increases. The normalized entropy is defined as

$$h(L) \equiv \frac{H\left(X_L\right)}{L}, \tag{4}$$

where we use the fixed length $L = 6$ in this paper.

The normalized entropy $h(L)$ measures the uncertainty of the random variable X_L and it is approximately proportional to the sum of $1/P(X_L)$. To investigate the properties of $h(L)$, we show the PDF $P(X_L)$ as a function of $X_L = (s_i, s_{i+1}, \dots, s_{i+L-1})$ for the sample ranges R for different periods.