Physica A 419 (2015) 349–355

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Emergence of bistable states and phase diagrams of traffic flow at an unsignalized intersection

Qi-Lang Li [a,*], Rui Jiang [b], Bing-Hong Wang [c,d]

[a] School of Mathematics and Physics, Anhui Jianzhu University, Hefei 230601, People's Republic of China

[b] School of Engineering Science, University of Science and Technology of China, Hefei 230026, People's Republic of China

[c] Department of Modern Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China

[d] Complex System Research Center, University of Shanghai for Science and Technology and Shanghai Academy of System Science, Shanghai 200093, People's Republic of China

HIGHLIGHTS

- The phase diagrams with complex topology structure are constructed.
- There exist bistable phases in some regions of the phase diagrams.
- The flow formulas in all regions in the phase diagram are derived.
- The deterministic Nagel and Schreckenberg model is adopted.
- Parallel update rules and yield dynamics are employed.

ARTICLE INFO

Article history:
Received 29 July 2014
Received in revised form 15 September 2014
Available online 18 October 2014

Keywords:
Traffic flow
Phase diagram
Intersection
Parallel update
Traffic state
Yield dynamics

ABSTRACT

This paper studies phase diagrams of traffic states induced by the bottleneck of an unsignalized intersection which consists of two perpendicular one-lane roads. Parallel updates rules are employed for both roads. At the crossing point, in order to avoid colliding, the consideration of yield dynamics may be suitable herein. Different from previous studies, the deterministic Nagel and Schreckenberg model is adopted in this work. Based on theoretical analysis and computer simulations, the phase diagrams of traffic flow have been presented and the flow formulas in all regions have been derived in the phase diagram. The results of theoretical analysis are in good agreement with computer simulation ones. One finds an interesting phenomenon: there exist bistable states in some regions of the phase diagrams.

1. Introduction

In the past few decades, vehicular flow has captured the interest of scientists [1–4]. As a typical many-body system composed of self-driven particles, vehicular flow exhibits diverse interesting non-equilibrium features such as capacity drop, first-order phase transition, metastable states and hysteresis, phase separation, self-organized criticality, spontaneous formation of traffic jams and wide scattering of statistical data in congested flow in the flow rate density plane [5–22].

To explain the features of vehicular flow, many classical and three-phase models (including cellular automata) have been proposed [2,3,18]. The two-phase traffic theory (see Refs. [4,20–23] and references therein) and Kerner's three-phase

* Corresponding author.
E-mail addresses: qilang@mail.ustc.edu.cn (Q.-L. Li), rjiang@ustc.edu.cn (R. Jiang), bhwang@ustc.edu.cn (B.-H. Wang).

http://dx.doi.org/10.1016/j.physa.2014.10.035
0378-4371/

traffic theory [18] assume that traffic flow is not always stable, and the complex traffic phenomena are related to traffic instability. Nevertheless, there are controversies of the two theories about (i) the fundamental assumption whether a unique relationship between flow rate and density exists or a two-dimensional region exists in the steady state, and (ii) traffic breakdown is associated with free flow to synchronized flow transition [18] or free flow to jam transition [4,20–23]. Recently more and more traffic data reveal the deficiency of two-phase theory. In particular, a recent experimental study of car-following behavior demonstrates that disturbances grow in a linear or concave way along the platoon, which runs against the two-phase theory. By removing the fundamental assumption in the two-phase car-following models and allowing the traffic state to span a two-dimensional region, the growth pattern of disturbances has changed qualitatively and becomes qualitatively or even quantitatively in consistent with that observed in the experiment.

The above mentioned controversy about traffic flow mainly arises in uninterrupted flow observed on highways, freeways or expressways. In urban traffic, intersections are fundamental operating units of road networks and have been widely studied in the literature. Since vehicles have to decelerate and accelerate frequently at the intersections, it is generally believed that urban traffic flow does not exhibit the various nonlinear phenomena as observed in uninterrupted traffic flow.

Phase diagrams are of great importance in thermodynamics with various applications in metallurgy, chemistry, physics, materials, etc. [24–28]. Nowadays, phase diagrams of traffic states induced by intersection bottlenecks have been frequently presented [29–35]. Some physicists have notably attempted to model and simulate the traffic flow at intersections including unsignalized and signalized intersections, roundabouts [29–39]. In the studies of unsignalized intersections [29–34], some scholars usually focus on a simple intersection which consists of two perpendicular one-lane roads: a northbound road and an eastbound one, which cross at the intersection point ($i = 0$) as shown in Fig. 1. For this crossing roads, two kinds of update rules are frequently adopted. In Refs. [33–35], the update is carried out for the northbound and eastbound roads in turn: in an odd (even) time step, all the vehicles on the northbound (eastbound) road are updated simultaneously. In Refs. [29–32], parallel update rules are employed. At each time step, all the vehicles on both roads are also updated simultaneously. At the intersection, if conflict occurs, the yielding dynamics will be implemented. Moreover, it is noted that the deterministic Fukui–Ishibashi (FI) is used to simulate the motion of vehicles in Refs. [29–35].

In a recent Letter [31], the authors have studied traffic flow at an unsignalized intersection, by using the FI model [40] and employing parallel update rules and yield dynamics. The phase diagram has been studied based on both theoretical analysis and computer simulations. It is shown that the phase diagram has several different topology structures for the cases of various maximum vehicle velocities. Nevertheless, the flow rate always changes continuously across the boundaries between different regions.

It is well-known that the acceleration of a vehicle is equal to its maximum velocity in the FI model, which is not very realistic. In 1992, Nagel and Schreckenberg (NS) have proposed a gradual way in which the velocity of a vehicle may increase by only one unit at most in one time step [41], which is different from the FI model in the acceleration rule. In this work, we present the deterministic NS model for the vehicle dynamics at an unsignalized intersection. Parallel update rules are adopted for the northbound and eastbound roads. At the crossing point, in order to avoid colliding, the consideration of yield dynamics may be suitable herein. Based on the theoretical analysis and computer simulations, we have mapped out the phase diagrams of traffic flow. Surprisingly, it is found that there exist bistable phases in some regions of the phase diagrams.

2. Description of the model

In the model, each road is composed of L cells of equal size and each cell can be either occupied by a vehicle with velocity $V = 0, 1, 2, \ldots, m$ or empty. Here m is maximum velocity of vehicles. The vehicles cannot turn at the intersection. The two roads intersect each other at the site $L/2$, see Fig. 1. In computer simulations, system size is set to $L = 2000$ and periodic boundary conditions are adopted.

We model the motion of vehicles by using the NS cellular automata traffic model. At each discrete time step $t \to t+1$, all the vehicles simultaneously update their states as follows: (I) Step 1: Acceleration; $v_i(t+1) \to \min(v_i(t)+1, m)$ on both roads. (II) Step 2: Deterministic deceleration to avoid accidents; $v_i(t+1) \to \min(v_i(t+1), d)$ on both roads. d is the number of empty cells in front of the ith vehicle. (III) Step 3 (only for approaching vehicles): if both approaching vehicles may arrive at the crossing point at $t+1$ time step, yield dynamics are implemented to avoid collision. One vehicle can occupy or pass the intersection and the other has to only move to the upstream site of the crossing point (denoted by $i = -1$ in Fig. 1). Let us denote the time needed for the approaching vehicle on the northbound (eastbound) road to arrive at the crossing point and its distance to the crossing point as t_n and d_n (t_e and d_e), respectively: (i) if $t_n > t_e$ ($t_n < t_e$), the movement priority is given to the eastbound (northbound) vehicle. (ii) If $t_e = t_n$, the movement priority is given to northbound (eastbound) vehicle provided $d_e > d_n$ ($d_e < d_n$). (iii) If $t_e = t_n$ and $d_e = d_n$, the movement priority is given to the two approaching vehicles with equal probability 0.5. (IV) Step 4: Movement; $x_i(t+1) \to x_i(t) + v_i(t+1)$, with $v_i(t)$, $x_i(t)$ being the velocity and the position of the ith vehicle at t time step, respectively.

3. Phase diagram in the general $m > 1$ case

Firstly we investigate the phase diagram in the $m > 1$ case. Fig. 2(a) presents the phase diagram under $m > 1$, which consists of six regions.