Physica A 419 (2015) 437–443

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Majority-vote model on a dynamic small-world network

Thomas E. Stone [a,b,*], Susan R. McKay [b]

[a] *Husson University, Bangor, ME, 04401, United States*

[b] *Department of Physics and Astronomy, University of Maine, Orono, ME, 04469, United States*

HIGHLIGHTS

- We study the robustness of social consensus to noise via the majority-vote model with noise on a dynamic small-world network.
- The critical behavior of the majority-vote model on a 2D dynamic small-world network is determined.
- Findings are consistent with the conjecture that dynamic and static network model share a universality class.

ARTICLE INFO

Article history:
Received 13 May 2014
Received in revised form 4 September 2014
Available online 18 October 2014

Keywords:
Majority-vote model
Dynamic small-world network
Universality
Finite-size scaling

ABSTRACT

Dynamic small-world networks combine short-range interactions within a fixed neighborhood with stochastic long-range interactions. The probability of a long-range link occurring instead of a short-range one is a measure of the mobility of a population. Here, the critical properties of the majority-vote model with noise on a two-dimensional dynamic small-world lattice are investigated via Monte Carlo simulation and finite size scaling analyses. We construct the order–disorder phase diagram and find the critical exponents associated with the continuous phase transition. Findings are consistent with previous results indicating that a model's transitions on static and dynamic small-world networks are in the same universality class.

1. Introduction

The majority-vote model (MVM) with noise [1,2] is one way to study the robustness of social consensus to noise (for this and other social dynamics models see the review in Ref. [3]). In the original formulation [1], each site on a square lattice can take one of two opinions, perhaps Democrat/Republican, yes/no, or up/down. In an update step, a randomly chosen site looks to its four orthogonally adjacent neighbors and adopts the majority opinion with probability $1 - q$ and the minority opinion with probability q, where q quantifies the noise. (If there is no minority, the site adopts each opinion with probability 1/2.) It was found that the lattice was disordered for $q > q_c$ but ordered for $q < q_c$, where the critical noise parameter was determined numerically to be $q_c = 0.075(1)$. This order–disorder transition piqued the interest of physicists, who have extensively studied the critical behavior of this transition on a number of topologies [4–13], as well as with a variety of modifications, such as different classes of agents [14], diffusion [15], damage spreading [16], and more than two opinions [17].

The critical behavior and, in particular, the universality associated with this model remain areas of active research. The original model [1] and a generalized version of that work [2] were both shown to fall into the equilibrium Ising universality class via the usual critical exponents β/ν, γ/ν, and $1/\nu$. However, of the other models mentioned above [4–17], only [14,16,17] are in the Ising class—the others tend to be in distinct classes governed by the underlying topology. For example,

* Corresponding author at: Husson University, Bangor, ME, 04401, United States.
E-mail address: stoneth@husson.edu (T.E. Stone).

http://dx.doi.org/10.1016/j.physa.2014.10.032
0378-4371/

an intriguing instance was the work by Campos et al. [6], which involved placing the MVM dynamics on top of a 2D small-world network. The small-world network [18] is very relevant from a social point of view, as it combines both short-range and long-range interactions, which is more realistic than either a regular lattice or a mean-field approach. It was shown in Ref. [6] that the critical exponents β/ν and γ/ν vary as a function of the concentration of long-range links. Here, we extend this previous work to a dynamic small-world (DSW) network in which the long-range links change with time, sometimes called a multiscale neighborhood model [19]. Specifically, our objective here is to characterize the order–disorder phase transition of the MVM with noise on a 2D DSW network. To this end, we construct the phase diagram in the $p-q$ plane, where p is a parameter that captures the amount of long-range interactions, and determine the critical exponents β/ν, γ/ν, and $1/\nu$ as functions of p.

2. Majority-vote model with noise

In the majority-vote model with noise, the ith site possesses an Ising spin variable $\sigma_i = \pm 1$. During an update step, a random site is chosen and the opinions (spins) of its neighbors are determined. With probability $1-q$ the site aligns with the majority while with probability q it aligns with the minority (hence q is usually called the noise parameter). The spin flip probability is then given by

$$w(\sigma_i) = \frac{1}{2}\left[1-(1-2q)\,\sigma_i S(x)\right] \tag{1}$$

where x is the sum of the spin variables of the neighbors to site i and $S(x) = \{-1, 0, 1\}$ for $\{x < 0, x = 0, x > 0\}$. A neighbor of the ith site is any other site that is connected to it in some fashion, which depends on the particular topology under consideration.

It is constructive to review the mean-field (MF) solution to this model since it indicates the presence of an order–disorder phase transition, and also because it will be a limiting case to the model under consideration in this work. First, define the usual site magnetization to be

$$m = \frac{1}{N}\sum_{i=1}^{N}\sigma_i \tag{2}$$

where N is the total number of sites. From master equation considerations [20,21] we can write

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\sigma_i\rangle = -2\langle\sigma_i w(\sigma_i)\rangle \tag{3}$$

for the time evolution of the average site magnetization. Now, for a coordination number of 4, $S(x)$ can be expressed as

$$S(x) = \frac{1}{8}(\sigma_1+\sigma_2+\sigma_3+\sigma_4)(3-\sigma_1\sigma_2\sigma_3\sigma_4) \tag{4}$$

where σ_j refers to the jth neighbor of site i [20]. Letting $m = \langle\sigma_i\rangle$ and substituting Eqs. (1) and (4) into Eq. (3) yields the mean-field time evolution of the magnetization

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -m + \frac{m}{2}(1-2q)(3-m^2) \tag{5}$$

$m^* = 0$ is always a fixed point of this equation, while $m^* = \pm\sqrt{\frac{1-6q}{1-2q}}$ are fixed points for $q < 1/6$ only. The derivative of the RHS of Eq. (5) is always negative for the latter fixed points, whereas it is negative for the $m^* = 0$ fixed point when $q > 1/6$ but positive for $q < 1/6$, indicating a supercritical pitchfork bifurcation at $q_c = 1/6$ [22]. Thus, in the mean-field limit we expect a order–disorder continuous phase transition at a critical noise of $q_c = 1/6$.

This order–disorder transition has been explored on a number of different topologies, including the square lattice [1,2,15], random graphs [4,5], scale-free networks [8,9], and small-world networks [6,7]. In this work we study the critical properties of this transition on a dynamic small-world network. DSW networks are similar to the more familiar static small-world (SSW) networks in that both contain a mixture of short and long range interactions that allow tuning between a completely regular lattice and an essentially random one. However, DSW networks generally have a fixed short-range neighborhood and a stochastic long-range neighborhood, with a rule specifying how a site will choose its links on a given time step. See Refs. [19,23–27] for a number of DSW formulations. While DSW networks capture the long-range/short-range essence of SSW networks and are a valid model in their own right, they can allow for significant computational speedup since large sparse adjacency matrices or link lists are avoided. Motivated by the multiscale neighborhood model presented in Ref. [19], we implement the MVM with noise dynamics on a DSW 2D square lattice as follows:

(i) Pick a random site to update, say the ith site.
(ii) Look at that site's four nearest neighbors successively. With probability $1-p$ use the spin of the nearest neighbor under consideration to update the sum of neighboring spins in Eq. (1). With probability p instead use the spin of a randomly chosen site to update the sum.
(iii) Flip σ_i in accordance with the spin flip probability $w(\sigma_i)$ given in Eq. (1).