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Protecting entanglement under depolarizing noise environment by using weak measurements

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HIGHLIGHTS

- An effective method for protecting entanglement in DP channel is proposed.
- Optimal entanglement can be obtained only by performing weak measurement on one qubit.
- The effect of our scheme is more pronounced in less initial entanglement.

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ABSTRACT

Weak measurements can effectively suppress amplitude-damping decoherence. In this paper, we study the effect of this method in protecting the entanglement of two qubits from independent depolarizing noisy channels. Our scheme consists of prior weak measurement on each qubit before the interaction with the noisy channel followed by a post recovering measurement. It is shown that the two-qubit entanglement can be enhanced to an optimal value by performing weak measurement and adjusting measurement parameter on one qubit. The effectiveness of our scheme is more pronounced in improving the two-qubit system with less initial entanglement. In addition, the maximal value of entanglement does not depend on the initial-state parameters.

1. Introduction

Quantum entanglement is not only a significant resource for quantum communication and quantum computation, but also a prominent feature which distinguishes the quantum realm from the classical one [1]. However, in realistic quantum information processing, decoherence [2–4], which is caused by the inevitably coupling of a quantum system with its external environment, leads to the degradation of quantum entanglement and in some certain cases, entanglement sudden death (ESD) [5–7]. Therefore, it is of great necessity to study the dynamics of quantum states exposed to noisy environment and devise effective ways to protect quantum entanglement from the detrimental effects of the environment. Several ways [8–11] for suppressing decoherence, such as decoherence-free subspace [8], entanglement distillation [9], and quantum Zeno dynamics [11] etc. have been proposed. Nevertheless, the above methods have their limitations more or less.

Recently, a new strategy, weak measurement, which is a generalization of Von Neumann measurements and associated with a positive-operator valued measure (POVM) [12], can be seen as a powerful tool for comprehending the basic concepts in quantum physics and has been found usefully in entanglement amplification [13,14]. Besides, it also can be utilized to

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well combat decoherence both for a single qubit and two-qubit system [15,16]. As far as we know, weak measurements have been experimentally realized in solid system [17], linear optic devices [15,18,19], and superconducting phase qubits [20,21], so it has attracted more and more people's attention [16,22–27]. Kim et al. [23] put forward an experimental scheme for protecting entanglement from amplitude-damping (AD) decoherence using weak measurement and quantum measurement reversal. Sun et al. [25] discussed a linear optics scheme for reversing the entanglement change due to weak measurement or amplitude damping of a two-qubit state. Man et al. [16] provided a scenario for manipulating entanglement of two qubits, which was stored in a common environment, via combined weak measurements and measurement reversals. Very recently, Wang et al. [26] proposed a program to protect quantum states from decoherence of generalized amplitude channel by means of weak measurement. However, we note that all the noise channels mentioned above are AD-class decoherence environments. To the best of our knowledge, nearly no study was pursued on how to suppress decoherence and amplify entanglement in depolarizing noise (DP) environment via methods of weak measurements.

Inspired by the above-mentioned schemes, in this work, we focus on how to suppress decoherence from depolarizing noise channel by using weak measurements. Our scheme consists of prior weak measurement on each qubit before the interaction with the noisy channel followed by a post recovering measurement. The result shows that the value of two-qubit entanglement can be enhanced to an optimal value only by performing weak measurement and choosing appropriate measurement parameter on one qubit. We also observe that the effectiveness of our scheme is more pronounced in improving the two-qubit system with less initial entanglement. In addition, it is interesting that the maximal value of entanglement does not depend on the initial-state parameters.

Our paper is organized as follows. In Section 2, we review some basic concepts, such as DP noise channel, concurrence, weak measurements and quantum measurement reversals. In Section 3, we analyze in detail the procedures for improving two-qubit entanglement in the DP noise environment by means of prior weak measurements and post measurement reversals, and also draw some conclusions. Section 4 ends with discussion of success probability and a brief summary.

2. Method

2.1. Depolarizing noise environment

Generally speaking, the initial pure states inevitably evolve into mixed states under the disturbance of noise environment. A depolarizing noise channel [1] can convert a qubit into a completely mixed state with probability p and keep it untouched with probability $1-p$. This process can be described by a quantum operation on the initial density matrix $\rho = |\psi(0)\rangle\langle\psi(0)|$,

$$\varepsilon_{DP}(\rho) = \sum_{i=0}^{3} E_i \rho E_i^{\dagger}. \tag{1}$$

The single qubit Kraus operators E_i [1] for DP noise are given by,

$$E_0 = \sqrt{1-p}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad E_1 = \sqrt{\frac{p}{3}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$
$$E_2 = \sqrt{\frac{p}{3}}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \qquad E_3 = \sqrt{\frac{p}{3}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{2}$$

where $p = 1 - \mathrm{e}^{-\gamma t}$ and γ is the decay factor of the DP noise. Since two qubits A and B are evolved independently, we have the following sixteen Kraus operators in terms of the tensor products of E_0, E_1, E_2, E_3:

$$\begin{aligned}
&K_1 = E_0^A \otimes E_0^B, \quad K_2 = E_0^A \otimes E_1^B, \quad K_3 = E_0^A \otimes E_2^B, \quad K_4 = E_0^A \otimes E_3^B,\\
&K_5 = E_1^A \otimes E_0^B, \quad K_6 = E_1^A \otimes E_1^B, \quad K_7 = E_1^A \otimes E_2^B, \quad K_8 = E_1^A \otimes E_3^B,\\
&K_9 = E_2^A \otimes E_0^B, \quad K_{10} = E_2^A \otimes E_1^B, \quad K_{11} = E_2^A \otimes E_2^B, \quad K_{12} = E_2^A \otimes E_3^B,\\
&K_{13} = E_3^A \otimes E_0^B, \quad K_{14} = E_3^A \otimes E_1^B, \quad K_{15} = E_3^A \otimes E_2^B, \quad K_{16} = E_3^A \otimes E_3^B.
\end{aligned} \tag{3}$$

2.2. Concurrence

The entanglement quality can be quantified conveniently by concurrence [28]. For a bipartite system ρ, the concurrence is defined as

$$C = \mathrm{Max}\{\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4, 0\} \tag{4}$$

where ε_i $(i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in descending order of the operator S, and $S = \rho(\sigma_1^y \otimes \sigma_2^y)\rho^*(\sigma_1^y \otimes \sigma_2^y)$. It is worth mentioning that under the standard product basis $\Re = \{|1\rangle \equiv |00\rangle, |2\rangle \equiv |01\rangle, |3\rangle \equiv |10\rangle, |4\rangle \equiv |11\rangle\}$, a simpler expression of concurrence for the two-qubit X-state can be written as

$$C = 2\max\left\{0, \sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}, \sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\right\} \tag{5}$$

where ρ_{ij} $(i, j = 1, 2, 3, 4)$ are the elements of density matrix ρ.