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A path-independent method for barrier option pricing in hidden Markov models

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HIGHLIGHTS

- We propose a path-independent method for pricing barrier options.
- We conduct an empirical study on barrier option pricing.
- We show that the proposed method is faster than Monte-Carlo simulation methods.

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ABSTRACT

This paper presents a method for barrier option pricing under a Black–Scholes model with Markov switching. We extend the option pricing method of Buffington and Elliott to price continuously monitored barrier options under a Black–Scholes model with regime switching. We use a regime switching random Esscher transform in order to determine an equivalent martingale pricing measure, and then solve the resulting multidimensional integral for pricing barrier options. We have calculated prices for down-and-out call options under a two-state hidden Markov model using two different Monte-Carlo simulation approaches and the proposed method. A comparison of the results shows that our method is faster than Monte-Carlo simulation methods.

1. Introduction

Barrier options are most popular exotic options. These financial instruments are mainly traded in over-the-counter markets. As mentioned in Ref. [1], "barrier option trading accounts for 50% of the volume of all exotic options and 10% of the volume of all traded securities". Barrier options pricing and hedging are important because many exotic options can be decomposed into barrier options. Barrier options have some features that become active when the underlying asset price crosses a predefined barrier. A barrier option can be an out or in option. An out option becomes worthless when the underlying asset price crosses a predefined barrier and the opposite is true for an in option. We call a barrier option an up (down) option if its barrier level is above (under) the initial stock price.

Researchers have proposed different methods and techniques for pricing barrier options under different market models. Merton presented the barrier option pricing formula under the Black–Scholes model [2]. It is well known that Black–Scholes model assumes that the underlying asset dynamics are described by a geometric Brownian motion with constant drift and volatility. However, some empirical studies such as Jizba et al. [3] and Liu et al. [4], Liu et al. [5] have shown that volatility changes of financial assets such as S&P 500 Index can be well fitted using certain probability distributions and that the

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volatilities of such assets are not constant. According to Fuha, Hub, and Lina [6], Black–Scholes model cannot describe three empirical phenomena: the asymmetric leptokurtic feature, the volatility smile, and volatility clustering. Several models have been proposed to consider these empirical phenomena. For example, ARCH [7,8], GARCH [8,9], multiscale GARCH [10], and stochastic volatility models such as Heston model [11] take time-dependent volatility into account.

One category of market models that recently has attracted researchers' interest is hidden Markov model. Hidden Markov models can describe the underlying asset dynamics more realistically than Black–Scholes model. Several empirical studies have provided evidence for the existence of different regimes in time series of asset and commodity prices. For example, Zhang and Wang [12] have shown that a significant two-state regime switching exists in WTI oil price; Weron, Bierbrauer, and Truck [13] have shown that regime-switching models can replicate the main characteristics of electricity spot price dynamics; and Schaller and Norden [14] have found a strong evidence for regime switching behavior in stock market return. According to Kilander [15], some of the benefits of employing a regime-switching framework are: (1) volatility clustering, (2) considering a heavy tailed distribution for the underlying asset payoff, (3) assuming different volatility structures, (4) consistency with modern financial theory, and (5) being more intuitive and much easier to understand than stochastic volatility models.

In hidden Markov models the parameters of underlying asset dynamics take different values according to a Markov process that describes the market state. The connection between hidden Markov models and statistical mechanics was first noticed by Sourlas [16]. Subsequently, Saul and Jordan [17] developed a statistical mechanical framework for discrete time series by which they related hidden Markov models to Boltzmann Machines that are a large family of exactly solvable models in statistical mechanics.

In recent years, there has been a considerable interest in pricing standard and exotic options in Markov switching framework (see Refs. [18,19] for barrier option pricing under Markov switching framework and see Refs. [20–24] for standard option pricing under regime switching). Hieber and Scherer [18] used Brownian bridge concept to present an efficient Monte-Carlo method for barrier option pricing in Markov switching framework. The advantage of their method is that no simulation of underlying asset dynamics between regime switching times is needed. The reason is that they used Brownian bridge concept to calculate the probability of barrier crossing between regime switching times. As a result, their method is fast and efficient. However, they need to check barrier hitting at regime switching times and at the maturity time for each randomly generated price path. Ching, Siu and Li [19] considered exotic option pricing when the underlying asset dynamics were described by a discrete time high order Markov model. They used Esscher transform to obtain an equivalent martingale measure. They also showed that considering high order Markov models would have an important impact on pricing path-dependent options such as barrier options. Yuen and Yang [20] modified the trinomial tree model of Boyle [25] in order to price standard and barrier options under Markov regime switching models. A disadvantage of their method is that the convergence rate is degraded when the volatilities of different regimes deviate largely from each other. Yuen and Yang [20] suggest completing the market by introducing the change of state contracts. Then, one would have to find risk neutral transition probability matrix which is hard to find as mentioned in Ref. [20]. Liu, Zhang and Yin [21] developed a fast Fourier transform (FFT) approach for standard option pricing in a Markov regime switching model. They obtained the Fourier transform of an option price in terms of the joint characteristic function of occupation times of the Markov chain. They also presented a near optimal FFT scheme when the underlying Markov chain has a large state space. However, it has been shown in Ref. [15] that a two-state regime switching model provides a very good in-sample pricing. Furthermore, increasing the number of states improves the performance of in-sample pricing slightly but the consumed time for estimating the model parameters becomes substantially longer. As shown in Ref. [15], the performance of out-of-sample pricing in regime switching models decreases when the number of states increases. Thus, as it is concluded in Ref. [15], a two state regime switching model would be sufficient for accurate option pricing. Buffington and Elliott [23,24] presented a method for pricing European and American options using partial differential equations. Elliott, Chan and Siu [22] presented a method for option pricing when the underlying risky asset would be driven by a Markov regime switching geometric Brownian motion. They assumed that the market parameters, for instance, the interest rate, the drift and the volatility of the underlying risky asset, would depend on unobservable states of the economy which could be modeled by a finite-state continuous-time hidden Markov process. They used a regime switching random Esscher transform to determine an equivalent martingale pricing measure so that the relative entropy between the equivalent martingale measure and the real world measure would be minimized. According to Greber and Shiu [26], Esscher transform is a transform that takes a probability density function and transforms it to new probability density function with parameter h that is called Esscher parameter. For the first time, Greber and Shiu [26] used Esscher transform, a time-honored tool in actuarial science, to determine an equivalent probability measure in an incomplete market. They showed how to determine Esscher parameter so that the discounted price of a security becomes a martingale under new probability measure. Regime switching random Esscher transform is different from standard Esscher transform in that the number of Esscher parameters in a regime switching Esscher transform is equal to the number of states in hidden Markov model. After determination of an equivalent martingale measure, Elliott et al. [22] used the method proposed in Refs. [23,24] to obtain the price of a European call option.

In this paper, we extend the option pricing method in Ref. [24] to price continuously monitored barrier options under a Black–Scholes model with regime switching. As in Ref. [22] we adopt a regime switching random Esscher transform to determine an equivalent martingale pricing measure. The regime switching Esscher parameter, and as a result the equivalent martingale measure, would stay the same because the underlying asset dynamics are not affected by the option type. Then, we show how to extend option pricing method in Refs. [23,24] to barrier option pricing. Finally, we derive a