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Stimulated Raman scattering in nonextensive statistics

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HIGHLIGHTS

- Stimulated Raman scattering (SRS) in a nonextensive plasma is analytically and numerically investigated.
- The generalized wavenumbers and growth rates are derived as a function of q-parameter.
- A relativistic Eulerian Vlasov code is used for comparing the analytical results with numerical simulations.

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ABSTRACT

Stimulated Raman scattering (SRS) in a collisionless plasma was analytically and numerically investigated in the context of nonextensive statistics proposed by Tsallis. The generalized wavenumbers and growth rates in both backward and forward scattering were derived as a function of different values of the nonextensive parameter q, which quantified the degree of nonextensivity of the system. It was shown that the increased numbers of superthermal particles and low velocity particles could affect the evolution of stimulated Raman scattering. A relativistic Eulerian Vlasov code for conditions similar to single-hot-spot experiments was used in order to compare the analytical results for the frequency and wavenumber with the quantities obtained from the numerical simulations. A nonextensive q-distribution by the restriction $q < 1$ was used as the initial velocity distribution. The results of numerical simulation showed a strong reflection by SRS. According to the simulation, the peak of the main components in both wavenumber and frequency spectra were found at positions estimated from the phase-matching conditions and the analytical data.

1. Introduction

Controlling parametric laser–plasma interactions is essential to the success of inertial confinement fusion (ICF) [1,2]. One of these instabilities is stimulated Raman scattering (SRS) [3,4], which involves the resonant decay of an incident electromagnetic wave (laser) into a scattered electromagnetic wave (EMW) and an electron plasma wave (EPW). SRS can reflect laser energy and preheat a target in inertial fusion experiments [5].

Most of the studies for SRS are derived in the frame of a fluid description, a Maxwellian distribution function, or a relativistic Maxwellian distribution function. However, many space and laboratory plasmas show a non-Maxwellian behavior [6–13]. Recently, there has been an increasing focus on a new statistical approach based on the generalization of the Boltzmann–Gibbs entropy, first recognized by Renyi [14] and subsequently suggested by Tsallis [15]. It is describedby a nonextensive parameter q, which specifies the degree of nonextensivity. For $q \neq 1$, it gives nonextensive q-distribution functions only when the parameter $q \to 1$ Maxwellian distribution is recovered. This nonextensive q-distribution function

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has been employed successfully in a wide range of systems characterized by nonextensivity for studying the dispersion property in plasmas [16–28].

In recent years, a family of so-called kappa velocity distributions introduced by Vasyliunas [29] has been recognized to be appropriate for studying the nonequilibrium plasma systems [30–37]. It is important to mention that because of a lack of formal derivation, a nonextensive approach to kappa distributions has been suggested [38] and a reduced kappa distribution, which can be transformed into a nonextensive q-distribution by the expression $\kappa = 1/(q-1)$, has been obtained [39–42]. The standard kappa distribution is recognized to be close to the nonextensive q-distribution; However, there is no simple transformation between the two distributions [43].

In this paper, we have analytically and numerically studied the effect of the deviation from a Maxwellian equilibrium (measured by the value of q-parameter) on the propagation of electron plasma waves, particularly focusing on the stimulated Raman scattering.

2. Theoretical model

Long time ago, Tsallis was inspired by heuristic arguments based on multifractals concepts to propose a generalization of the Boltzmann–Gibbs (B–G) entropy formula for statistical equilibrium [15,44–47]. Recently, nonextensive statistical mechanics has become a useful tool for describing complex systems whose properties cannot be exactly described by considering B–G statistical mechanics. In Tsallis statistics, the entropy has the form of [15]

$$S_q = k_B \frac{1 - \sum_i p_i^q}{q-1} \tag{1}$$

where k_B is the standard Boltzmann constant, p_i is the probability of the ith microstate, and q is a parameter quantifying the degree of nonextensivity. The B–G entropy is recovered in the limit $q \to 1$. The basic property of Tsallis entropy is the non-additivity or nonextensivity for $q \neq 1$. For example, for two systems A and B, the rule of composition reads

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \tag{2}$$

In the nonextensive framework, the three-dimensional equilibrium distribution function, $f_0(v)$, is given by [48]

$$f_0(v) = n_0 B_q \left[1 - (q-1)\frac{v^2}{v_T^2}\right]^{1/(q-1)} \tag{3}$$

where the normalization constant reads

$$B_q = \frac{\sqrt{1-q}}{\pi^{3/2} v_{T\sigma}^3} \left(\frac{3q-1}{2}\right) \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)}, \quad \text{for } -1 < q \leq 1 \tag{4}$$

and

$$B_q = \frac{\sqrt{q-1}}{\pi^{3/2} v_{T\sigma}^3} \left(\frac{3q-1}{2}\right) \left(\frac{q+1}{2}\right) \frac{\Gamma\left(\frac{1}{q-1} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{q-1}\right)}, \quad \text{for } q \geq 1 \tag{5}$$

where n_0 is the number of electrons per unit volume of the plasma, $v_T = \sqrt{2k_BT/m}$ is the thermal velocity of electrons, and $\Gamma(x)$ is the well-known Euler gamma function. Taking into account that $\lim_{|z|\to\infty}[\Gamma(a+z)/\Gamma(z)]z^{-a} = 1$ [49], it is easy to see in the limit $q \to 1$ that the three-dimensional Maxwellian distribution function is obtained. It is important to mention that in the case $q > 1$, the distribution function has a thermal cutoff, namely $v_{cut\text{-}off} = \sqrt{v_T^2/(q-1)}$.

In the recent years, the q-nonextensive velocity distributions have been used in the framework of the kinetic plasma theory. For example, in 2000, Lima et al. [16] derived the dispersion relation for electron plasma waves in a collisionless thermal plasma in the context of nonextensive statistics. They assumed that the distribution function of electrons was modified by a perturbation signal, while the distribution function of ions could be considered as an invariable quantity. However, Chen et al. [50] showed that the results obtained by Lima et al. were not appropriate and they drived the correct dispersion relation and landau damping for electron plasma waves.

The properties of electron plasma waves (EPWs) are important for investigating stimulated Raman scattering, so in this study the propagation of EPW in a nonextensive plasma was investigated.

For the longitudinal propagating in an unmagnetized, collisionless plasma with a background of fixed ions, the dispersion relation can be written as

$$D(\omega, k) = 1 + \frac{\omega_{pe}^2}{k^2} \int d\mathbf{v} \frac{\mathbf{k} \cdot \frac{\partial \hat{f}_0(\mathbf{v})}{\partial \mathbf{v}}}{\omega - \mathbf{k} \cdot \mathbf{v}} = 0 \tag{6}$$