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Flocking of multi-agent systems with multiplicative and independent measurement noises

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HIGHLIGHTS

- We investigate the Cucker–Smale type system with independent and multiplicative measurement noise.
- The flocking behavior depends on the intensity of noise and the group density.
- We report that, in noise environments, it is rather easy for the group with high density to form and maintain ordered group motion.
- We find that, under the perturbation of multiplicative noise, system with weak coupling can tolerate relatively strong perturbation of noise.

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ABSTRACT

We investigate the effect of noise and group density on the emergence of flocking in a stochastic Cucker–Smale type system. We consider the independent and multiplicative measurement noise and derive sufficient conditions for flocking and non-flocking in terms of the noise intensity and group density. We report that, in noise environments, it is rather easy for the group with high density to form and maintain ordered group motion. Moreover, we find that, under the perturbation of multiplicative noise, systems with weak coupling can tolerate relatively strong perturbation of noise.

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1. Introduction

Flocking is a kind of collective phenomenon often found in a group of interacting individuals, which is widespread in natural and man-made systems. Examples include the emergent behavior of birds [1], schooling of fishes [2,3], foraging of ant colonies [4], swarming of bacteria [5,6] and locusts [7]. As a significant collective behavior, it has attracted more and more attention due to its broad applications in many fields ranging from engineering [8–13] to sociology [14]. Despite ubiquity and importance of flocking behavior, we still know little about the mechanism of such emerging behavior. There has been a lot of interesting research on flocking problems. In 1995, Vicsek et al. proposed a simple self-propelled particles model in which two distinct agents interact if and only if they are within a preassigned distance [15]. In Vicsek model, the authors postulated that each particle adjusts its velocity in response to the mean of the relative velocities of its near neighbors. The numerical studies of Vicsek model reveal a phase transition depending on the group density. Its analytic behavior was subsequently studied by Jadbabaie et al. [16]. Based on Vicsek's model and Jadbabaie's work, Cucker and Smale introduced a simple dynamical model which assumes that each particle adjusts its velocity in response to a weighted average of the

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difference of its velocity with those of the other particles. For convenience, we will call it C–S model. The core assumption of the C–S model is that each individual influences all of the other, no matter what the configuration of the individuals [1]. Different from the Vicsek model, the convergence results of C–S model depend on conditions of the initial state only, which is the main virtue of Cucker and Smale's work. In Ref. [1], the authors showed that if the communication has a long-range interaction, then a global unconditional flocking occurs, while for the short-range interaction the conditional flocking occurs for some restricted class of initial configurations. Later several extensions of the C–S model have been proposed and analyzed [17,18]. For more details, see the survey of Ref. [19] and references cited therein.

In reality, communication processes are inevitably subjected to noise and disturbance in the environment. Recently, the effect of noise on the communication processes has received considerable attention [20–26]. The first investigation of flocking in noise environments was done by Cucker and Mordecki [20], who added random noise to the C–S model and found that flocking occurs in finite time with a certain confidence. In Ref. [21], Ha et al. studied the stochastic C–S model with additive noise, which means that noises additively input the communications. The authors showed that the total velocity error of C–S system is uniformly bounded if the communication rate satisfies a lower bound condition. The stochastic flocking of a C–S model with multiplicative white noise was investigated in Refs. [22,23]. In Ref. [22], the authors showed that if the noise is weak enough and the communication rate satisfies a lower bound condition, then the flocking can take place asymptotically. In contrast, under the effect of strong noise on particles, the flocking does not occur. However, the noise in Refs. [22,23] is one dimensional, which means that all agents are subject to the same perturbation. In real systems, e.g. bird flocks and fish schools, each agent is always perturbed by independent stochastic factors [27,28]. In addition, we can observe the phenomenon that the flocking in the group with small size usually fails due to the noise perturbation. In contrast, the group with large size can transit to ordered group motion rapidly, in spite of the noise. However, the mechanism that the group with large size can tolerate strong perturbation of noise remains poorly understood.

The main purpose of this paper is to investigate the influence of noise and group density on the emergence of flocking in a C–S type model with independent and multiplicative measurement noise. We incorporate noise into the C–S model by taking into account that, under the perturbation of noise, each individual cannot accurately measure the states of all of the other individuals. The intensity of noise on each agent is proportional to the difference of its velocity with those of the other individuals. First, we investigate the condition for the emergence of flocking. Then, we consider the non-flocking problem that the flocking might fail by increasing the intensity of noise. In this paper, we present sufficient conditions for flocking and non-flocking which depend on the noise intensity and group density. We verify that the proposed model exhibits a phase transition between flocking and non-flocking depending on the intensity of noise, the group density and the coupling strength between individuals. Our results agree with the fact that it is rather easy for the group with high density to form and maintain ordered group motion. Our results also indicate that the group with weak coupling can tolerate relatively strong perturbation of noise.

The rest of the paper is organized as follows. In Section 2, we present a C–S type model with independent and multiplicative measurement noise, and give the definition of mean-square flocking. The sufficient conditions for flocking of the proposed model is presented in Section 3. The non-flocking problem is discussed in Section 4. Some numerical examples are given in Section 5. Finally, some conclusions are given in Section 6.

2. Problem statement

Consider a model consisting of N autonomous agents. Let $(x_i, v_i) \in R^2$ be the position and velocity of ith agent. The C–S model can be described by the following equations:

$$\begin{cases} \dot{x}_i = v_i, \quad 1 \le i \le N, \\ \dot{v}_i = \dfrac{1}{N} \displaystyle\sum_{j=1}^{N} \psi(|x_j - x_i|)(v_j - v_i). \end{cases} \tag{1}$$

Here, the communication rate $\psi(\cdot) : [0, \infty) \to [0, \infty)$ quantifies the influence between ith and jth particles and is a non-increasing function. In Ref. [1] and most of subsequent papers, ψ is taken as

$$\psi(y) = \frac{K}{(1 + y^2)^\beta}.$$

In noise environments, the agent usually cannot measure its neighbor's states exactly. The measurement of state of jth agent received by ith agent can be modeled by

$$y_{ij}(t) = v_j(t) + \sigma_{ij}\big[v_j(t) - v_i(t)\big]\xi_{ij}, \tag{2}$$

where $\xi_{ij} = \{\xi_{ij}(t), t \ge 0\},\ i, j = 1, 2, \ldots, N$ are standard independent white noises, $\sigma_{ij} \ge 0$ represent the intensities of noise. For simplicity, we assume that $\sigma_{ij} = \sigma_{ji}, \forall i, j = 1, 2, \ldots, N$. By (2), when $j = i$, $y_{ii}(t) = v_i(t)$, which indicates that agent i can measure its state exactly.

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