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A visibility graph averaging aggregation operator

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HIGHLIGHTS

- A novel aggregation operator is proposed based on the visibility graph.
- The proposed visibility graph averaging (VGA) operator conserves time information.
- The weights are decided by the distribution of the degree in the visibility graph.
- The VGA operator is used to analyze TAIEX database and shows that it is effective.

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ABSTRACT

The problem of aggregation is of considerable importance in many disciplines. In this paper, a new type of operator called visibility graph averaging (VGA) aggregation operator is proposed. This proposed operator is based on the visibility graph which can convert a time series into a graph. The weights are obtained according to the importance of the data in the visibility graph. Finally, the VGA operator is used in the analysis of the TAIEX database to illustrate that it is practical and compared with the classic aggregation operators, it shows its advantage that it not only implements the aggregation of the data purely, but also conserves the time information. Meanwhile, the determination of the weights is more reasonable.

1. Introduction

Aggregation is a process of combining several numerical values into a single one which exists in many disciplines, such as image processing [1,2], pattern recognition [3,4], decision making [5–7] and so forth [8–13]. To obtain a consensus quantifiable judgments, some synthesizing functions have been proposed.

For example, arithmetic mean, geometric mean, median can be regarded as a basic class, because they are often used and very classic. However, these operators are not able to model an interaction between criteria. For having a representation of interaction phenomena between criteria, Fuzzy measures have been proposed by Sugeno in 1974 [14]. Two main classes of the fuzzy measures are Choquet and Sugeno integrals. Choquet and Sugeno integrals are idempotent, continuous and monotone operators. The ordered weighted average (OWA) operators is a particular case of discrete Choquet integrals. The OWA operators were introduced by Yager in Ref. [15] to provide an aggregation which lies in between the "and" and the "or" operators. The "and" (t-norms) and the "or" (t-conorms) operators are generalizations of the logical aggregation operators which are two specialized aggregation families. Above operators try to look for giving a "middle value", but the t-norms and the t-conorms can compute the intersection and union of fuzzy sets.

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However, to the best of our knowledge, these operators do not consider the influence of time specially and the time factor should not be ignored in some areas such as economics, space science, weather forecast and so forth. In this paper, a novel aggregation operator called visibility graph averaging (VGA) aggregation operator is proposed which can aggregate the time series effectively.

This paper is inspired by the pioneering work the visibility graph [16] which builts a natural bridge between complex network theory and time series. In the visibility graph, the values of a time series are plotted by using vertical bars. These vertical bars are regarded as landscapes. Every bar is linked with others that can be seen from the top of the considered one, then the associated graph is obtained. According to the study, it is found that the structure of the time series is conserved in the graph topology. For example, periodic series convert into regular graphs, random series convert into random graphs, and fractal series convert into scale-free graphs. Until now, the visibility graph has been applied in economics [17], geology [18,19], praxiology [20], biological system [21,22] and so forth [23–28]. The proposed visibility graph averaging (VGA) operator is based on the visibility graph. Hence, it conserves the time information likewise.

In some aggregation operators, how to decide the weight of each argument is a problem [29–31], but in this proposed VGA operator, while the time series is converted into graphs, the degree distribution is decided. Meanwhile, the weights are decided. In general, if the degree of a node is bigger than others, this node will be more important, and in the visibility graph a node represents a data value in the time series, so it offers a reasonable way to determine the weights of the corresponding data values.

The remainder of this paper is organized as follows. Section 2 briefly introduces some necessary preliminaries of the aggregation operators and the graph theory. Section 3 details the proposed visibility graph averaging (VGA) aggregation operator. The properties of the visibility graph averaging (VGA) aggregation operator will be discussed in Section 4. In Section 5, VGA aggregation operator is applied in economics and compared with OWA operators to show its advantage. Finally, some conclusions are given in Section 6.

2. Preliminaries

In this section, the aggregation operator and the graph theory are briefly introduced.

2.1. The aggregation operator

Aggregating values, a new value can be obtained, but this can be done in different ways. In other words, aggregation operators is various. In the following, the aggregation operator will be introduced in a general way.

Let R be the set of real numbers. $A_j(x)$ ($A_j(x) \in R$) denotes the degree to which x satisfies the criteria A_j, $D(x)$ ($D(x) \in R$) denotes the set of the results of the aggregation.

Definition 1. An aggregation operator is a function Agg:

$$D(x) = Agg(A_1(x), A_2(x), \ldots A_n(x))$$

where n represents the number of values to be aggregated.

Several fundamental conditions have been proposed to define the aggregation operators [32]. The fundamental properties which generalize most of the precedent definitions are as follows Ref. [33]:

(1) *Identity when unary*: If there is only one value needing to be aggregated, the result is itself.

$$Agg(A_j(x)) = A_j(x).$$

(2) *Boundary conditions*: If all the values needing to be aggregated are completely bad, false or not satisfactory, the result has to be completely bad, false or not satisfactory. On the contrary, if all the values needing to be aggregated are completely good, true or satisfactory then the result has to be completely good, true or satisfactory.

$$Agg(0, 0, \ldots, 0) = 0$$
$$Agg(1, 1, \ldots, 1) = 1.$$

(3) *Monotonicity*: If the individual value increases the overall satisfaction should increase:

$$Agg(A_j(x)) \geq Agg(A_j(y)), \quad \text{if } A_j(x) \geq A_j(y).$$

2.2. The graph theory

Graph theory is the study of graphs, which is made up of vertices and edges. Graphs can be used to deal with many types of relations and processes in computer science [34], biological [35,36], social [37] and so forth [38–40]. Recently, with the development of complex network research, the graph theory is widely used in the analysis of complex network [41–43].

Definition 2. A graph is formed by vertices and edges connecting the vertices.