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Multiscale multifractal detrended cross-correlation analysis of financial time series

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HIGHLIGHTS

- Discuss the cross-correlation between two stock markets in both multiscale and multifractal view.
- Expend J. Gieraltowski's MMA method to two time series and apply the new process to financial time series for the first time.
- Discuss the influence of the length of series to the multiscale and multifractal results.
- Using the artificial time series to proving the efficiency of our multiscale multifractal detrended cross-correlation analysis.

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ABSTRACT

In this paper, we introduce a method called multiscale multifractal detrended cross-correlation analysis (MM-DCCA). The method allows us to extend the description of the cross-correlation properties between two time series. MM-DCCA may provide new ways of measuring the nonlinearity of two signals, and it helps to present much richer information than multifractal detrended cross-correlation analysis (MF-DCCA) by sweeping all the range of scale at which the multifractal structures of complex system are discussed. Moreover, to illustrate the advantages of this approach we make use of the MM-DCCA to analyze the cross-correlation properties between financial time series. We show that this new method can be adapted to investigate stock markets under investigation. It can provide a more faithful and more interpretable description of the dynamic mechanism between financial time series than traditional MF-DCCA. We also propose to reduce the scale ranges to analyze short time series, and some inherent properties which remain hidden when a wide range is used may exhibit perfectly in this way.

1. Introduction

Financial markets are considered as complex dynamic systems [1–5]. Many empirical studies have investigated financial data scaling behavior [6–9] and several models have been proposed to account for the observed multifractal features [10]. One of the important features of market dynamics is the presence of cross-correlations between financial variables. Podobnik and Stanley [11] extended the detrended fluctuation analysis (DFA) method introduced by Peng et al. [6] into two time series in 2008, and proposed the detrended cross-correlation analysis (DCCA), which is widely used for discussing the cross-correlations between financial variables [12–21]. The study of cross-correlations between stock time series is attractive for both fundamental and practical researches [22–24]. On the fundamental side, a financial market is regarded as a complex system consisted by many interacting components. A sudden changing on a certain component can be affected by the other

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elements as well as itself. On the practical side, the feature of cross-correlations is important for evaluating the risk of an investment in the stock market. Therefore, it is of great interest in quantifying the cross-correlation characteristics.

However, many sequences do not exhibit a simple monofractal behavior, instead, multifractal behavior is often observed. Simply speaking, a time series is multifractal if its fluctuations of different magnitude have different scaling exponents, and such a wide range of scaling exponents is preserver for every reasonably long part of the time series. Kantelhardt et al. [25] interpreted this multifractal phenomenon by two reasons: one is the broad probability density function for the values of the series, another one is the different long-range correlations of the small and large fluctuations. They also developed the DFA method for analyzing the multifractal characteristics, named multifractal detrended fluctuation analysis (MF-DFA). Naturally, Zhou [26] extended this idea for two time series for the purpose of obtaining the multifractal features in the power-law cross-correlations between two time series or higher-dimensional quantities recorded simultaneously, and named it multifractal detrended cross-correlation analysis (MF-DCCA). Since then, a lot of authors have taken part in the analysis of multifractal properties between the time series. Some of them discussed the effects of different trends on MF-DCCA and proposed a variety of different MF-DCCA methods using different trend filtering methods [27,28]. In the empirical analysis side, Wang et al. [29] investigated the cross-correlations between Chinese *A*-share and *B*-share markets using MF-DCCA method and found multifractality exists. Zhao et al. [30] suggested that multifractal cross-correlation features are significant in traffic signals using the multifractal Fourier detrended cross-correlation analysis method.

In some cases, though, there exist crossover scales separating regimes with different scaling exponents, e.g. long-range correlations on small scales and another type of correlations or uncorrelated behavior on large scales. In other cases, the scaling behavior is more complicated [31], and different scaling exponents are required for different parts of the series [25]. Hence, a single exponent cannot fully describe the dynamics of signals [32]. It has been proposed to estimate a short- as well as a long-term exponent to better describe the signal's dynamics. Always, this two coefficient model is used in most studies of physiology variability. While no matter single or two coefficient models, they are both viewed as a simplification of a more complex phenomenon. So, some authors began to propose new methods, which are based on DFA for estimating a continuous spectrum of scale exponents [33–36]. In this way they can describe the fractal property changing continuously with the time scale.

More recently, it was demonstrated many times that the fractal properties vary from point to point along the series, on a whole new field of vision, Gieraltowski et al. [37] introduced a method called multiscale multifractal analysis (MMA), which describes the heart rate variability to a new extension of including the dependence on the magnitude of the variability and time scale. The MMA method may provide new ways of measuring the nonlinearity of a signal, and it may help to develop new methods for the study of dynamical features of the series. Here we extend the MMA method to the analysis of cross-correlation properties between two time series. We will discuss the multifractal behavior of the cross-correlations between two sequences at multiple scales and apply it to the study of stock market time series.

The structure of this article is as follows: In Section 2, we describe the methods we use in this article. In Section 3, we list the stock market time series and the artificial test series we used to demonstrate our method. In Section 4, we show the obtained results. We give a summary in Section 5.

2. Methods

We introduce the method of multifractal detrended cross-correlation analysis (MF-DCCA) first, and then extend it to multiscale multifractal detrended cross-correlation analysis (MM-DCCA), which is the new method we develop.

2.1. Multifractal detrended cross-correlation analysis

Assume that $x(i)$ and $y(i)$, $(i = 1, 2, \ldots, N)$ are two time series, where N is the length of the series, then the MF-DCCA method can be summarized as follows:

Step 1: Describe the "profile" of each series:

$$X(i) = \sum_{k=1}^{i}(x_k - \langle x\rangle), \qquad Y(i) = \sum_{k=1}^{i}(y_k - \langle y\rangle), \quad i = 1, \ldots, N$$

where $\langle x\rangle = \frac{1}{N}\sum_{k=1}^{N} x_k$ and $\langle y\rangle = \frac{1}{N}\sum_{k=1}^{N} y_k$.

Step 2: Divide X and Y into $N_s = [N/s]$ non-overlapping segments of equal length s. Since the record length N is often not a multiple of the considered time scale s, a short part at the end of each profile will remain in most cases. In order not to disregard this part of the series, the same procedure is repeated starting from the other end of each profile. Thus, $2N_s$ segments are obtained together.

Step 3: Calculate the local trends for each of the $2N_s$ segments by a least-square fit of each series. Then we calculate the difference between the original time series and the fitting polynomial.

$$F^2(s, v) = \frac{1}{s}\sum_{i=1}^{s} |X((v-1)s + i) - X^v(i)| \cdot |Y((v-1)s + i) - Y^v(i)|$$