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City traffic jam relief by stochastic resonance

F. Castillo [a], B.A. Toledo [a,*], V. Muñoz [a], J. Rogan [a,b], R. Zarama [c,d], M. Kiwi [a,b], J.A. Valdivia [a,b,d]

[a] Departamento de Física, Facultad de Ciencias, Universidad de Chile, Santiago, Chile
[b] CEDENNA, Santiago, Chile
[c] Departamento de Ingeniería Industrial, Universidad de los Andes, Bogotá, Colombia
[d] CEIBA complejidad, Bogotá, Colombia

HIGHLIGHTS

- We simulate a row of interacting cars using a cellular automaton model.
- The jammed state dynamics is analyzed in a sequence of synchronized traffic lights.
- Small density jammed states show the expected scaling laws close to the resonance.
- A stochastic resonance-like behavior is found when we include velocity perturbations.

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ABSTRACT

We simulate traffic in a city by means of the evolution of a row of interacting cars, using a cellular automaton model, in a sequence of traffic lights synchronized by a "green wave". When our initial condition is a small density jammed state, its evolution shows the expected scaling laws close to the synchronization resonance, with a uniform car density along the street. However, for an initial large density jammed state, we observe density variations along the streets, which results in the breakdown of the scaling laws. This spatial disorder corresponds to a different attractor of the system. As we include velocity perturbations in the dynamics of the cars, all these attractors converge to a statistically equivalent system for all initial jammed densities. However, this emergent state shows a stochastic resonance-like behavior in which the average traffic velocity increases with respect to that of the system without noise, for several initial jammed densities. This result may help in the understanding of dynamics of traffic jams in cities.

Transport problems represent an interesting field due to its high social impact and its emergent properties [1–4]. Of particular importance are traffic and pedestrian flows, that have been studied extensively in the past [5–10]. In these systems, behaviors such as jamming transitions and chaos have been found to be common [11–13]. Here, we consider the traffic in a city as represented by a number of interacting cars moving through a sequence of traffic lights, a system which has many nontrivial features [14–21].

For example, Varas et al. [21] showed that the critical behavior found around the synchronization resonances [11] is robust, not only with respect to the street length sequence, but also with respect to the car density for an unjammed initial condition. The study of the system was characterized for two traffic light phase behaviors: (a) synchronized phases [14], and (b) phases linked by a green wave [11]. Resonance occurs when (a) the traveling time between traffic signals is the same

* Corresponding author. Tel.: +56 02 7763322.
E-mail address: btoledo@macul.ciencias.uchile.cl (B.A. Toledo).

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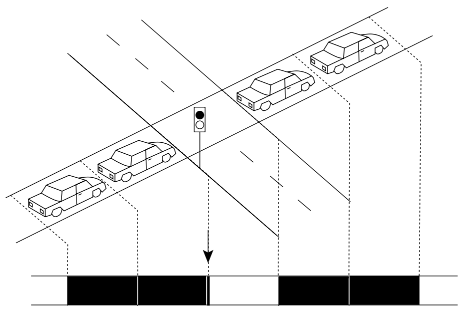

Fig. 1. Possible system state, with a car stopped at a traffic light with a long traffic jam ahead, in which case the car will not move, even if the signal is green. A schematic representation of the state is shown in the lower part of the figure.

as the period of the signals in the synchronized phase strategy, and (b) when the average speed of the car is the same as the green wave velocity in the green wave strategy. In this manuscript we will consider the green-wave synchronization resonances, due to its popular applicability in cities. These resonances are the boundary between two different dynamics, but as was shown numerically and analytically for a single car [11,21], the behavior close to the resonance does not depend on the finite braking and accelerating capabilities of the cars. Recently, the very complex spatiotemporal phenomena of self-organization that occur under green-wave propagation have been studied in detail using a discrete version of the Kerner–Klenov stochastic three-phase microscopic model for large and small flow rates [22]. The complex behavior determined some of the physical effects associated with green-wave propagation. However, that model does not stress explicitly the coupling effect of the traffic light on the car flow, so that the dynamics close to the synchronization resonances was not studied. In a series of published manuscripts, where we have considered the very fine details of the effect of the traffic lights on the car evolution by including continuous accelerating and braking capacities, we have shown that close to the synchronization resonance the car behavior does not depend on the intricacies of the acceleration [11,15]. Hence, the purpose of present manuscript is to investigate how the dynamics close to the synchronization resonance changes as we increase the initial number of interacting cars that jam the system. For that purpose we simulate the dynamics of a number of cars by a simple cellular automaton (CA) model. A large number of CA variants have been proposed to simulate city traffic, including many details of the car dynamics [5]. But, for the purpose of this work, we concentrate on a very simplified CA model since, as mentioned above, the critical behavior close enough to the synchronization resonance should be more or less insensitive to these details (*e.g.*, finite braking and accelerating capabilities, etc. [11,14]).

We will show below that this deterministic CA model displays different behavior close to resonance, depending on the initial jammed density. The different behaviors, representing different attractors of the system, are produced by the spatial variation of the jammed density at each traffic light. As we introduce velocity perturbations in the system, we note that the average velocity increases with respect to the system without velocity perturbations, in a type of stochastic resonance that is produced by the time average homogenization of the spatial variation of the jammed density at each traffic light. This stochastic resonance is similar, although not equal, to the standard stochastic resonance [23,24] observed in the amplification of signals by noise [25], bi-stable nonlinear systems [26,27], climate transitions [28–30], biological systems [31,32], etc.

Following Ref. [21], we consider a street of length L_{tot} with N_s traffic lights. The length L_n between the nth and $(n+1)$th traffic light is divided in $N_{L_n} = L_n/\ell$ cells of length ℓ. The time it takes a car to move to the next cell, namely τ, is the automaton evolution time step. A car will move to the next cell only (a) if no other car is stopped in the next cell; and (b) if the current cell has a traffic light, it must be in green and the next two cells must be empty, so that the drivers avoid stopping at the intersection at the following cell.

The only possible values for the velocity of the cars are $v_{\max} = \ell/\tau$, which corresponds to one cell per time step, and 0. We are also assuming that the cars cannot pass each other. Fig. 1 shows a schematic representation of a state of the system at a particular time. Occupied cells are represented by a black block. An arrow over the cell boundary represents a traffic light.

The switching of the nth traffic light, from green to red and vice-versa, is given by the periodic function $f_n(t) = \sin(\omega_n t + \phi_n)$. When $f_n(t) > 0$ the traffic light is green, and the cars at the intersection can move to the next unoccupied cell. If $f_n(t) < 0$ a red traffic light stops the motion of the vehicles approaching to it. Here, ω_n represents the frequency of the nth traffic light, although for simplicity we are considering that all traffic lights have the same period T, *i.e.* $\omega_n = 2\pi/T\ \forall n$. For the case of the green wave strategy studied here, a green pulse propagates through the sequence of traffic lights with velocity v_{wave}, so that $\phi_n = -(\omega/v_{\mathrm{wave}})\sum_{j=0}^{n} L_j$. We define $\alpha = v_{\max}/v_{\mathrm{wave}}$, to compare the green wave speed with the cars maximum speed. If we take that (a) the distance between traffic lights is about $L = 200$ m, representative of the Alameda Av. in Santiago, Chile; (b) the length of each cell is $\ell = 10$ m; the cruising velocity is $v_{\max} = 10$ m/s (36 km/h); then each time step corresponds to $\tau = 1$ s. Let us note that these values are consistent with the car having equal accelerating and braking capabilities of $a = v_{\max}^2/2\ell = 5$ m/s^2. For the traffic light period we will use $T = 60$ s.

The dynamics described above corresponds to a nontrivial modification of the car model presented in Ref. [21] as we consider an initial jammed situation that shows more complexity than the empty road analyzed in that reference. From