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Entanglement of a 2-qubit system coupled to a bath of quantum spin glass

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HIGHLIGHTS

- Quantum entanglement (concurrence) of a 2-qubit system coupled to a spin glass bath of 2 to $n \geq 4$ qubits.
- A general formula is obtained to describe the concurrence with $J = 0$ and $h = 0$ for n bath sites.
- The physics of a 2-qubit system coupled with n bath sites for $J = 0$ is analytically described.
- For small fluctuation in J, a mean steady state average concurrence of about 0.5 is obtained.

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ABSTRACT

We study the quantum entanglement (concurrence) of a 2-qubit system coupled to a small spin glass bath of 2 to $n \geq 4$ qubits. The bath is described by the quantum *XX* Heisenberg model with random J coupling and varying magnetic field h. We look at the dynamics of the steady state average concurrence for the system and obtain a general formula to describe the concurrence with $J = 0$ and $h = 0$ for n bath sites. The physics of 2-qubit system coupled with n bath sites for $J = 0$ is analytically described for small n. The result for large n was numerically found to be qualitatively similar. For small fluctuation in J, a mean steady state average concurrence of about 0.5 is obtained.

1. Introduction

The study of phase transitions in spin glass remains one of the most challenging problems in condensed matter physics. A spin glass consists essentially of a random collection of spins or magnetic moments frozen spatially and temporally. The disorder and randomness in the spins typically result in some frustration in the interaction energy due to the competing interactions between the spins caused by the random mixture of the ferromagnetic and antiferromagnetic couplings or the random positions of the spins [1–6]. As a result of this frustration, the ground state of a spin glass in general becomes a disordered state embedded in a complex and rugged free energy landscape. Spin glasses are produced by introducing impurities like manganese (Mn), iron (Fe), europium (Eu), etc. with a host which can in general be any non-magnetic metal that dissolve the impurities. Examples of spin glasses are alloys of copper and manganese, $Cu_{1-x}Mn_x$ [7] or gold and iron, $Au_{1-x}Fe_x$ [8]. Other alloys possessing insulating or conducting properties can also be made into spin glasses. Examples are europium strontium sulfur $Eu_xSr_{1-x}S$ [9] which is a semiconductor and lanthanum gadolinium aluminum $La_{1-x}Gd_xAl_2$ [10] which is a metal.

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Experimentally, the magnetic susceptibility of spin glass materials shows a cusp at a certain freezing temperature T_f for low applied magnetic field. The cusp in the magnetic susceptibility at certain freezing temperature suggests that there may be a phase transition in the materials [11]. It has been shown that with just 100 G of applied magnetic field, the cusp is destroyed producing a broader maxima [8,12,13]. A number of theories have been developed to describe the phase transition observed experimentally. In the Edwards–Anderson (EA) model [14], the spins only interact with the nearest neighbor couplings and there is hardly any long range order. In the Sherrington–Kirkpatrick (SK) model [15], every spin couples equally with every other spin. Despite the success of these models for describing some spin-glass behavior, the models in general do not seem to explain fully all the observations in experiments. Moreover, new insights and methods developed in this field have been found to be useful in other areas of condensed matter [6,5,16]. In recent years, a quantum spin glass model of the form $LiHo_xY_{1-x}F_4$ has been extensively studied both experimentally and theoretically [17–22]. The magnetic Ho^{3+} ions in these materials behave like effective Ising spins while the yttrium Y^{3+} are non-magnetic ions.

In quantum information theory, entanglement [23–29] in spin chains have been studied extensively. The interest stems from the fact that entanglement is a key resource for quantum information processing. The latter is believed to be able to perform a number of tasks more efficiently than its classical counterpart. Entanglement have found applications in quantum key distribution, quantum dense coding, quantum teleportation, entanglement swapping and others [27]. The theory of entanglement has also been applied to many-body systems where the zero and finite temperature properties of entanglement and how entanglement is closely connected to the characteristics of the phase diagram [29] is studied. However, entanglement alone does not account for all the non-classical aspect of quantum correlations [27].

While nearest-neighbor entanglement is important for quantum computing, the possibility for remote entanglement is also one of the major challenges in eventual realization of quantum computers. In nature, all systems decohere. Decoherence is a natural process in which a quantum state interacts with its environment and tends towards a statistical mixture of states [30,31]. Hence, a good understanding of decoherence, and the role of entanglement lies at the heart of quantum information processing. In the past few years, studies on decoherence due to entanglement between the system and the environment have been carried out extensively [32–36]. Most of these studies focus on understanding how a ferromagnetic, antiferromagnetic or paramagnetic quantum spin bath environment interacts with one or two-qubit system at thermal equilibrium. Lai et al. [36] has investigated the reduced dynamics of a single or two qubits coupled to an interacting quantum spin bath modeled by a *XXZ* spin chain with the use of time-dependent density matrix renormalization group (t-DMRG). Recently, Winograd et al. [37] have studied the decoherence of a single qubit weakly coupled to a frustrated spin bath. They consider the cases in which the bath is described by a classical spin glass and a quantum random transverse Ising model. Although much research have been done on understanding these system–environment interaction, there has been relatively few work done on the interaction between a system with a quantum spin glass bath with uniform applied magnetic field. It is also not known how such quantum states of a two-qubit system will evolve over time.

With this motivation, we examine and describe the behavior of the interaction between a two-qubit system and a quantum spin glass bath environment where the initial state of the system is prepared as a separable state. Specifically, in this paper, we numerically investigate a two-qubit system, coupled to a quantum spin glass bath. Using this model, we examine how a bath of a non-spin glass (NSG) or a spin glass (SG) interacts with the system by varying the standard deviation of the coupling (between bath sites) and the external magnetic field (h).

The paper is organized as follows. We begin in Section 2 by defining the Hamiltonian of the 2-qubit system coupled to a quantum spin bath environment of three qubits. The spin bath is essentially modeled as a spin glass exhibiting the usual characteristics of disorder and frustration. We then discuss the quantum correlation (concurrence) between the 2-qubit system, after tracing out the spin bath environment as a function of the external applied magnetic field and the standard variation in the coupling numerically. These results are described and presented in Section 3. In Section 4, we present some analytical forms of the model for the special case of zero coupling (between the spin bath sites) and magnetic field. We also study the steady state behavior of the average concurrence. In Section 5, we summarize our results indicating some of the limitations of our work and some possible future directions.

2. Theoretical formulation

In this section, we describe the Hamiltonian used in our study. After formulating the Hamiltonian of the system and environment, we evolve the initial state of the system and environment over time. In order to look at the quantum correlation of the 2-qubit system, we trace out the environment and compute the entanglement measure, i.e. concurrence, of the 2-qubit system. The formalism is general and the environment can be extended to an n-qubit system. The *XX* Heisenberg spin glass is described by

$$H_n = \sum_{i=1}^{n} J_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) \tag{1}$$

where J_i are random variables and σ_i^α denotes the Pauli matrices ($\alpha = x, y, z$) of the ith spin [29] which is subject to the periodic boundary condition $\sigma_{i+1}^\alpha = \sigma_1^\alpha$. In this case, the ith spin represents the individual site number in the bath environment. The exchange energies J_i are quenched random variables with a probability distribution $P(J_i) = \frac{1}{\sqrt{2\pi\Delta}} e^{-J^2/2\Delta^2}$