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The phase diagram and the pathway of phase transitions for traffic flow in a circular one-lane roadway

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ABSTRACT

This paper demonstrates that patient driving habits lead to homogenous congested flow while impatient driving habits lead to wide-moving jam flow in the high density region based on the numerical simulation of the intelligent driver model proposed by M.Treiber [M. Treiber, A.Hennecke, D. Helbing, Phys. Rev. E 62 (2) (2000), 1805–1824]. In a circular one lane traffic system which includes homogeneous drivers, we obtain the stable condition of homogenous flow and the phase diagram of traffic flow based on the linearization analysis. The phase diagram shows three possible pathways of phase transition along with the increase of global density: from the homogenous free flow to the homogenous congested flow directly, from the homogenous free flow to the synchronized flow then to the homogenous congested flow, or from the homogenous free flow to synchronized flow then to the wide-moving jam flow. The paper also analyzes the traffic flow including heterogenous drivers, and the results indicate that homogenous congested flow will lose its stability when the proportion of impatient drivers reaches a critical value and some new kinds of traffic flow emerge: wide-moving jam flow or a mixture of synchronized flow and wide-moving jam flow.

1. Introduction

Looked as a typical complex system containing many self-driven interacting bodies, the traffic flow has been investigated by statistical and nonlinear methods for decades. A community of physicists have taken great efforts to understand the different phases of traffic flow, such as free flow at low density regions, synchronized flow at intermediate density regions, and jammed flow at higher density regions, and the mechanism of transition between different phases [1–8]. As shown by many empirical data, many macroscopic traffic flow models and some microscopic car-following traffic models, the traffic flow is a kind of free flow with low global density and evolves into homogenous congested flow with high global density. However, Kerner and his collaborators discovered "synchronized flow" and wide-moving jam flow based on empirical data [1,8–10]. Therefore, there are several possible paths from homogenous free flow to jammed flow along with the increase of global density. In order to solve this problem, Kerner had proposed a "three phase theory" implemented by a piecewise function of acceleration and incorporated the response of the driver into a time delay acceleration model [9]. In this paper, we will tackle the same problem using an intelligent driver model. This model is an easy to calibrate, accident free car following model. It describes what drivers actually do when they drive and already reproduces some empirical data [2].

The basic microscopic mechanism of traffic flow is the driver's behavior which has more and more influence on the traffic flow state with the increases of global density. It is easy to imagine that different driving habits will lead to different

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types of traffic flow even if the global density is identical. Therefore, we consider two kinds of driving habits: patient and impatient driving. Based on the stability analysis of the intelligent driver model and numerical simulation, we show that patient driving habit leads to homogenous congested flow and impatient driving habit results in wide-moving jam flow if the global density is high. At the same time, we work out the phase diagram of traffic flow and propose the possible routes of transition between different traffic phases. Then we discuss the effects of a heterogenous driver on the behavior of traffic flow. Finally, we come to the conclusion and discussion.

2. The model

We follow the intelligent driver model and consider N vehicles running on a closed circular one-lane roadway of length L. The nth vehicle's driver adjusts the speed according to the velocity v_n, the relative velocity with respect to its leading car $\Delta v_n = v_{n+1} - v_n$, and the headway $s_n = x_{n+1} - x_n$ (where x_n is the coordinate of the nth vehicle, and $x_{N+1} = x_1$, $v_{N+1} = v_1$) as follows [5]:

$$\dot{v}_n = a_n \left\{ 1 - \left(\frac{v_n}{v_{0,n}} \right)^{\delta} - \left(\frac{s_n^*}{s_n} \right)^2 \right\} \tag{1}$$

where a_n is the maximum acceleration, $v_{0,n}$ is the desired velocity of the nth vehicle, δ is the acceleration exponent, and s_n^* is the minimum desired headway which varies dynamically with the velocity and the relative velocity:

$$s_n^* = s_{0,n} + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}} \tag{2}$$

where $s_{0,n}$ is the jam distance that must be kept at least even if the traffic flow comes to a complete standstill, T_n is the safe time headway, and b_n is the desired deceleration. We set the parameters as follows: $N = 150$, $v_{0,n} = v_0 = 20$ m/s, $a_n = a = 0.8$ m/s^2, $b_n = b = 1.8$ m/s^2, $l_n = l = 5$ m, and $\delta = 4$.

3. Simulation of patient and impatient driving habits

The intelligent driver model has a trivial homogenous flow, whose velocity v_h is determined by $s_h = \frac{s_0 + T v_h}{\sqrt{1 - (\frac{v_h}{v_0})^{\delta}}}$, and whose flux is $Q_h = \rho v_h$, where $\rho = \frac{N}{N(l+s_h)} = \frac{1}{l+s_h}$ is global density. If the homogenous flow loses its stability, the velocity of a vehicle will depend on the time. We denote standard deviation of velocity by $\sigma = \sqrt{\frac{1}{N}\sum(v_n - \bar{v})^2}$, where $\bar{v} = \frac{1}{N}\sum_n v_n$, and the relative deviation of velocity by $r = \frac{\sigma}{\bar{v}}$. Let Q denote flux of traffic flow, and it may be different from that of homogenous flow, and the ratio $q = \frac{Q}{Q_h} = \frac{\bar{v}}{v_h}$ measures the difference.

In the intelligent driver model, the jam distance $s_{0,n}$ and safe time T_n reflect the driving habit of the nth driver. Different driving habits will cause different dynamic behavior of traffic flow. Here we investigate two different driving habits: patient and impatient driving. Imagine that one driver plunges into a traffic jam; the driver must brake the car with deceleration a_{brake} to avoid a collision with its leading vehicle:

$$a_{brake} \sim \frac{s_{0,n}}{T^2}. \tag{3}$$

The patient driver needs a relative smaller deceleration to brake the vehicle while the impatient driver must have a relative larger deceleration to brake the vehicle.

To get an intuitional impression about the effect of driving habits on the traffic flow, we carry out the numerical simulation and show the results in Fig. 1. In order to simulate the traffic flow, we use the discrete form of Eqs. (1) and (2): $x_n(t + \Delta t) = x_n(t) + v_n(t + \Delta t)\Delta t$, $v_n(t + \Delta t) = max(0, v_n(t) + \dot{v}_n(t)\Delta t)$, where $\Delta t = 0.1s$. We assume that a patient driver has a headway $s_0 = 1.5$ m, and safe time $T = 2$ s while the impatient driver has a headway $s_0 = 1.5$ m, and safe time $T = 1.2$ s. As shown in Fig. 1, traffic flow of patient drivers and impatient drivers exhibit different behaviors along with the increase of global density. At the low density region, $r = 0$ and $q = 1$ hold for traffic flows composed by patient or impatient drivers, which means the traffic flow is homogenous free flow. When the global density is greater than a critical value $\rho_{1,\gamma}(\gamma = patient, impatient)$, homogenous flow loses its stability, the flux of traffic flow becomes smaller than that of homogenous flow ($q < 1$), and the relative deviation of a vehicle quickly increases to a certain value ($r > 0$). Before the density increases to another critical value $\rho_{2,\gamma}$, the driving habit has no remarkable influence on the behavior of traffic flow. The vehicles slow down but still move on. It can be verified that traffic flow is a kind of synchronized flow or mixture of synchronized flow with wide-moving jam flow. The driving habit leads to quite different dynamical behaviors after the density reaches $\rho_{2,\gamma}$. The homogenous flow reobtains (the flow became homogenous but congested) the stability habit since $r = 0$, $q = 1$ hold for high density if the drivers are patient. But r and q increase sharply in the case of impatient drivers, which implies that speed of vehicle changes quickly and the flux is much bigger than that of homogenous flow. Fig. 1c shows the velocity of traffic flow produced by patient and impatient drivers with high global density ($\rho > \rho_{2,\gamma}$). It is easy to see that a patient driving habit leads to a congested homogenous flow, and the impatient driving habit leads to a wide-moving