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Traffic states and fundamental diagram in cellular automaton model of vehicular traffic controlled by signals

Takashi Nagatani

Department of Mechanical Engineering, Division of Thermal Science, Shizuoka University, Hamamatsu 432-8561, Japan

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ABSTRACT

We present a cellular automaton (CA) model for vehicular traffic controlled by traffic lights. The CA model is not described by a set of rules, but is given by a simple difference equation. The vehicular motion varies highly with both signals' characteristics and vehicular density. The dependence of tour time on both cycle time and vehicular density is clarified. In the dilute limit of vehicles, the vehicular motion is compared with that by the nonlinear-map model. The fundamental diagrams are derived numerically. It is shown that the fundamental diagram depends highly on the signals' characteristics. The traffic states are shown for various values of cycle time in the fundamental diagram. We also study the effect of a slow vehicle on the traffic flow.

1. Introduction

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic lights to give priority for a road because the city traffic networks often exceed the capacity. Recently, transportation problems have attracted much attention in the fields of physics [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [1–30]. Interesting dynamical behaviors have been found in the transportation system. The jams, chaos, and pattern formation are typical signatures of the complex behavior of transportation [24,25].

Brockfeld et al. have studied optimizing traffic lights for city traffic by using a CA traffic model [31]. They have clarified the effect of signal control strategy on vehicular traffic. Also, they have shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model [32]. They have derived the relationship between the road capacity and jamming transition. Until now, one has studied the periodic traffic controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [31,32]. Very recently, Lammer and Helbing have studied the vehicular flow by the self- control of traffic signals in urban road networks [33].

In real traffic, the vehicular traffic depends highly on the configuration of traffic lights and the priority of roadways. In the dilute limit of vehicular density, a few works have been carried out for the traffic of vehicles moving through an infinite series of traffic lights with the same interval. The effect of cycle time on vehicular traffic has been clarified by using the

E-mail address: tmtnaga@ipc.shizuoka.ac.jp.

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nonlinear-map models [34–38]. Also, it has been shown that the inhomogeneity of a signal's interval and irregular split have important effects on vehicular traffic [39].

The nonlinear-map model is an effective tool for the vehicular traffic in the dilute limit in which there are no interactions between vehicles. However, the city traffic is affected by both signals' characteristics and vehicular density. The interaction between vehicles changes with both signal and vehicular density. Therefore, the vehicular traffic controlled by traffic lights changes with its density. There is little known about how the dependence of tour time on the cycle time changes with vehicular density. The vehicular traffic controlled by signals will show a complex behavior. It will be necessary and important to study the vehicular traffic controlled by signals, using a simple dynamic model. In real traffic, the fundamental diagram is very important. It is meaningful to know the relationship between the fundamental diagram and cycle time. Furthermore, when the slow vehicle is introduced in city traffic, it is important to know how the slow vehicle affects the traffic flow.

In this paper, we study the vehicular traffic through a series of traffic lights for various values of vehicular density. We present a deterministic cellular automaton model for the vehicular traffic controlled by signals. The CA model is described by the simple difference equation. We investigate the dynamical behavior of vehicular traffic by using the difference equation. We clarify the dynamical behavior of the vehicular traffic through a sequence of signals, by varying both cycle time and vehicular density. We show how the dependence of the tour time on the cycle time changes with the vehicular density. We compare the vehicular traffic with that obtained by the nonlinear-map model. Also, we study the effect of a slow vehicle on the traffic flow.

2. CA model and difference equation

We consider the flow of vehicles going through the series of traffic lights on a one-dimensional lattice. Each vehicle does not pass over other vehicles. The traffic lights are positioned homogeneously on a roadway. The interval between signals has a constant value and is given by l. We consider the synchronized strategy for the signal control. In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ (s_pt_s). The period of green is s_pt_s and the period of red is $(1-s_p)t_s$. Time t_s is called the cycle time and fraction s_p represents the split which indicates the ratio of green time to cycle time. We set split as $s_p = 0.5$.

We extend the deterministic CA model proposed by Fukui and Ishibashi [1,28,40] to take into account traffic lights. We define the position of vehicle i at time t as $x_i(t)$ where x, i, and t are an integer. The CA model of Fukui and Ishibashi is given by

$$x_i(t+1) = \min[x_i(t) + v_{\max}, x_{i+1}(t) - 1], \tag{1}$$

where $v_{\max}$ is the maximum velocity and an integer. Here, $\min[A, B]$ is a minimum function and takes the minimum value within A and B. The velocity takes the integer value ranging from 0 to $v_{\max}$. The velocity depends on the headway. If headway $\Delta x_i(t)$ $(=x_{i+1}(t)-x_i(t))$ is larger than the maximum velocity, the vehicle moves with the maximum velocity. If the headway is less than the maximum velocity, the vehicle moves with velocity $\Delta x_i(t) - 1$.

When a vehicle arrives at a traffic light and the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and the traffic light is green, the vehicle does not stop and goes ahead without changing speed. The position of the closest signal before vehicle i at time t is given by

$$x_{i,s}(t) = \left\{\text{int}\left(\frac{x_i(t)}{l}\right) + 1\right\} l. \tag{2}$$

Then, the position of vehicle i at time $t+1$ is given by

$$\begin{aligned} x_i(t+1) = {} & \min[x_i(t) + v_{\max}, x_{i+1}(t) - 1]\{1 - \vartheta(\sin(2\pi t/t_s))\} \\ & + \min[x_i(t) + v_{\max}, x_{i+1}(t) - 1, x_{i,s}(t) - 1]\vartheta(\sin(2\pi t/t_s)). \end{aligned} \tag{3}$$

Here, $\vartheta(t)$ is the step function. It takes one if $t > 0$ and zero if $t \leq 0$. When there are no signals, Eq. (3) reduces to Eq. (1). If the signal just before vehicle i is green, $\vartheta(\sin(2\pi t/t_s)) = 0$ and $x_i(t+1) = \min[x_i(t)+v_{\max}, x_{i+1}(t)-1]$. Eq. (3) reduces to Eq. (1). Otherwise, if the signal just before vehicle i is red, $\vartheta(\sin(2\pi t/t_s)) = 1$ and $x_i(t+1) = \min[x_i(t)+v_{\max}, x_{i+1}(t)-1, x_{i,s}(t)-1]$. Then, if the headway is larger than $v_{\max}$, vehicle i stops at site $x_{i,s}(t) - 1$ just before the signal. Also, if $x_{i+1}(t)$ is higher than $x_{i,s}(t)$, vehicle i stops at site $x_{i,s}(t) - 1$ just before the signal. Thus, Eq. (3) presents the CA model for the vehicular traffic through a series of traffic lights. Eq. (3) is a single difference equation. Until now, the CA model for the signal traffic has been described by a set of CA rules. However, model (3) is of great advantage to simulate because the difference equation is simple.

It will be expected that the vehicular traffic exhibits a complex behavior. We study how the vehicular traffic changes by varying both cycle time and vehicular density.

3. Nonlinear-map model

In the dilute limit of vehicular density, the interaction between vehicles is negligible. Vehicles do not interfere with each other. Then, it is sufficient only to consider a motion of a single vehicle through a series of traffic lights. Here, we consider the