Physica A 439 (2015) 44–47

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Ermakov–Ray–Reid systems with additive noise

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HIGHLIGHTS

- Euler–Maruyama numerical scheme with additive noise displayed for Ermakov systems.
- Ermakov–Lewis invariant is found less robust than in the case of multiplicative noise.
- Similar results are obtained for a more general Ermakov–Ray–Reid system.

ARTICLE INFO

Article history:
Received 25 March 2015
Received in revised form 15 July 2015
Available online 30 July 2015

Keywords:
Ermakov–Lewis invariant
Additive noise
Euler–Maruyama method
Forced parametric oscillator
Ermakov–Ray–Reid system

ABSTRACT

Using the methods developed by us in Cervantes-López et al. (2014) for multiplicative noises, we present results on the effects of the additive noise on the Ermakov–Lewis invariant. This case can be implemented in the Euler–Maruyama numerical method if the additive noise is considered as the forcing term of the parametric oscillator and presented as a particular case of the Ermakov–Ray–Reid systems. The results are obtained for the same particular examples as for the multiplicative noise and show a tendency to less robustness of the Ermakov–Lewis invariant to the additive noise as compared to the multiplicative noise.
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This work is the second paper in the series concerned with the effects of noises on the stochastic parametric oscillators that we started with Ref. [1], where we focused on the effects of multiplicative noises. Here we investigate the robustness of the Ermakov–Lewis (EL) invariant to additive noises by making usage as in Ref. [1] of the Euler–Maruyama discretization for the numerical calculations. As well known, the standard Ermakov system refers to the motion of a free parametric oscillator

$$\ddot{x} + \Omega^2(t)x = 0, \tag{1}$$

together with the associated Milne–Pinney nonlinear equation

$$\ddot{\rho} + \Omega^2(t)\rho = \frac{k}{\rho^3}, \tag{2}$$

where k is an arbitrary real constant. The solutions of (1) and (2) can be related through the following formula [2]

$$x(t) = C\rho(t)\sin(k\Theta_T(t) + \phi), \tag{3}$$

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http://dx.doi.org/10.1016/j.physa.2015.07.023
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Table 1
The invariants for the case (ii) with their mean values and standard deviations for the three cases of $\Omega(t)$ considered here, each of them in the presence of weak, intermediate, and strong noise amplitudes.

		I ($\rho^2\dot{f}$ included)	I ($\rho^2\dot{f}$ not included)
$\Omega(t)=2$,	$\alpha_\Omega=0$	1.000000 ± 0.000000	1.000000 ± 0.000000
	$\alpha_\Omega=0.001$	1.001280 ± 0.000770	1.001280 ± 0.000770
	$\alpha_\Omega=0.01$	1.012550 ± 0.007800	1.012640 ± 0.007801
	$\alpha_\Omega=0.1$	1.104260 ± 0.089765	1.104990 ± 0.089832
$\Omega(t)=2\sin t$,	$\alpha_\Omega=0$	1.000000 ± 0.000000	1.000000 ± 0.000000
	$\alpha_\Omega=0.001$	1.000370 ± 0.000805	1.000380 ± 0.000805
	$\alpha_\Omega=0.01$	1.003790 ± 0.008051	1.003920 ± 0.008051
	$\alpha_\Omega=0.1$	1.046310 ± 0.081424	1.047570 ± 0.081271
$\Omega(t)=2t^2$,	$\alpha_\Omega=0$	1.000000 ± 0.000000	1.000000 ± 0.000000
	$\alpha_\Omega=0.001$	1.001140 ± 0.001017	1.001140 ± 0.001017
	$\alpha_\Omega=0.01$	1.011460 ± 0.010115	1.011450 ± 0.010116
	$\alpha_\Omega=0.1$	1.116590 ± 0.097451	1.116500 ± 0.097468

where C and ϕ are arbitrary constants and the total phase $\Theta_T(t)$ is given by

$$\Theta_T(t)=\int^t\frac{1}{\rho^2(t')}\mathrm{d}t'. \tag{4}$$

Such parametric systems are endowed with the EL invariant given by

$$I_0=\frac{kx^2}{2\rho^2}+\frac{1}{2}\left(\dot{x}\rho-\dot{\rho}x\right)^2. \tag{5}$$

Here we show that the effects of the additive noise can be evaluated by extending the Ermakov system to the forced case, as worked out for example in Ref. [3] and in the more general context of forced Ermakov–Ray–Reid systems in Refs. [4–8]. In particular, we consider the following forced parametric oscillator as presented in Ref. [4]

$$\ddot{x}+\Omega^2(t)x=f(t)+\frac{1}{x^2\rho}g(\rho/x), \tag{6}$$

with $g(\rho/x)$, an arbitrary external force of the Ray–Reid type, for which the EL invariant reads

$$I=I_0+\dot{\psi}x-\psi\dot{x}+\int^t\psi(\tau)f(\tau)\,\mathrm{d}\tau-\rho^2xf(t)+\int^{\rho/x}g(\tau)\mathrm{d}\tau, \tag{7}$$

where the function $\psi(t)$ is the solution of a second auxiliary equation

$$\ddot{\psi}(t)+\Omega^2(t)\psi(t)=\rho^2\dot{f}(t)+3\rho\dot{\rho}f(t)+\frac{1}{x^3\rho}g(\rho/x)\,\psi. \tag{8}$$

It is important to mention that in this case the Milne–Pinney equation (2) keeps its form unchanged [4]. As in Ref. [1], we will write the dynamical systems given by the Eqs. (6), (2), and (8) as a matrix version of a stochastic matrix differential equation of the form

$$\mathrm{d}Y_t=a(t,Y_t)\mathrm{d}t+b(t,Y_t)\mathrm{d}B_t, \tag{9}$$

where B_t is the stochastic variable, for which the Euler–Maruyama numerical method is readily available [9–11]. In the matrix formulation, the corresponding stochastic variables and coefficients are identified in the following explicit forms

$$\mathrm{d}X_t=\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}\dot{x}\end{pmatrix},\qquad a(t,X_t)=\begin{pmatrix}\dot{x}\\ -\Omega^2x+\dfrac{g}{x^2\rho}\end{pmatrix},\qquad b(t,X_t)=\alpha_\Omega\begin{pmatrix}0\\ 1\end{pmatrix}, \tag{10}$$

$$\mathrm{d}\rho_t=\begin{pmatrix}\mathrm{d}\rho\\ \mathrm{d}\dot{\rho}\end{pmatrix},\qquad a(t,\rho_t)=\begin{pmatrix}\dot{\rho}\\ -\Omega^2\rho+\dfrac{1}{\rho^3}\end{pmatrix},\qquad b(t,\rho_t)=\begin{pmatrix}0\\ 0\end{pmatrix}, \tag{11}$$

$$\mathrm{d}\psi_t=\begin{pmatrix}\mathrm{d}\psi\\ \mathrm{d}\dot{\psi}\end{pmatrix},\qquad a(t,\psi_t)=\begin{pmatrix}\dot{\psi}\\ \rho^2\dot{f}+\dfrac{g\psi}{x^3\rho}-\Omega^2\psi\end{pmatrix},\qquad b(t,X_t)=3\alpha_\Omega\begin{pmatrix}0\\ \rho\dot{\rho}\end{pmatrix}, \tag{12}$$

where α_Ω is the amplitude of the noise [12]. In Ref. [1], the simplest case of multiplicative noise, $m=1$, has been illustrated and a strong robustness of the Ermakov–Lewis invariant has been reported. In the present calculations, the term $\rho^2\dot{f}$ in $a(t,\psi_t)$ has been disregarded because of its negligible effects on the mean values and the standard deviations of the invariant as seen in Table 1. The chosen initial conditions are as in Ref. [1], i.e., $x(0)=1, \dot{x}(0)=0, \rho(0)=1, \dot{\rho}(0)=0$, and additionally, $\psi(0)=1, \dot{\psi}(0)=0$.

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