Physica A 439 (2015) 66–74

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Heider balance, asymmetric ties, and gender segregation

Małgorzata J. Krawczyk [a], Marcelo del Castillo-Mussot [b], Eric Hernández-Ramírez [b], Gerardo G. Naumis [b], Krzysztof Kułakowski [a,*]

[a] AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Kraków, Poland

[b] Instituto de Física, Universidad Nacional Autónoma de México, 04510 México D.F., Mexico

HIGHLIGHTS

- Theory of Heider balance is generalized for the case of asymmetric ties.
- The gender segregation is evaluated from the data on 37 Mexican school classes.
- A new index of gender segregation is proposed and statistically validated.

ARTICLE INFO

Article history:
Received 25 February 2015
Received in revised form 13 May 2015
Available online 3 August 2015

Keywords:
Networks
Dynamics
Communities
Gender

ABSTRACT

To remove a cognitive dissonance in interpersonal relations, people tend to divide their acquaintances into friendly and hostile parts, both groups internally friendly and mutually hostile. This process is modeled as an evolution toward the Heider balance. A set of differential equations have been proposed and validated (Kułakowski et al., 2005) to model the Heider dynamics of this social and psychological process. Here we generalize the model by including the initial asymmetry of the interpersonal relations and the direct reciprocity effect which removes this asymmetry. Our model is applied to the data on enmity and friendship in 37 school classes and 4 groups of teachers in México. For each class, a stable balanced partition is obtained into two groups. The gender structure of the groups reveals stronger gender segregation in younger classes, i.e. of age below 12 years, a fact consistent with other general empirical results.

1. Introduction

The gender segregation is a well-known phenomenon which shapes our social life from the preschool years up to puberty [1]. It becomes visible already between children of age less than three years. Estimations of frequency of overall pro-social attitude of children of 33 months old indicate that it increases systematically when related to the same-sex children: for boys (from 19 vs. girls to 29 vs. boys) and girls (from 17 vs. boys to 31 vs. girls) [2]. When children grow, the effect is reinforced; the proportion of time spent with the same sex to cross-sex partners increases from 3:1 for the children of 4.5 years to 11:1 for the children of 6.5 years [3]. While the segregation is well documented, its origin remains disputable; for an extensive discussion we refer the reader to Refs. [1–5] and references cited therein. In particular, cognitive factors have been distinguished as one of the possible driving forces toward the segregation [1,2]; this approach assumes that children (even very young) are able to distinguish sex of their coplayers and their own. Another possible explanation is that boys and

* Corresponding author.
E-mail address: kulakowski@fis.agh.edu.pl (K. Kułakowski).

http://dx.doi.org/10.1016/j.physa.2015.07.027
0378-4371/

girls have different styles of play, respectively more aggressive and more passive, and they prefer to contact with those of the same style. As none of these theories has been confirmed in experimental studies [2,4,6], the issue remains open; yet the segregation persists under different forms in adulthood [5]. What is undisputed is that it is a group effect, i.e. children use sex (or gender) as a criterion of social grouping [1]. We note that after [1], we use the words *sex* and *gender* interchangeably.

Here we are interested in a general method of detection of the gender segregation by means of data analysis. We assume that the input data are given in the form of a matrix, which contains information on preferences of agents of a given group toward other members of this group. Data in this form has been collected and analyzed many times, with the paper of Zachary on the karate club [7] as a canonical example. The new element here is that we allow for asymmetric relations, i.e. the feeling of person A about person B is not necessarily the same as the feeling of B about A. In other words, the related matrix is asymmetric, and the related networks of relations are directed.

Focusing on social relations, when designing an appropriate method of data analysis, our starting point is the model of the change of perception: the removal of cognitive dissonance [8–11]. As the outcome, the method provides the group partition into two mutually hostile parts, with friendly relations within each part. This state is equivalent to a consistent partition of each triad of agents into a pair of friends and their common enemy; alternatively, all three ties in the triad are friendly. The state is known as the Heider balance [12,13]. Per analogy to an Ising spin glass [14], in the balanced state there is no frustrated triads; the product of bonds is positive for each triad.

Our model approach [8,10,11] applies a set of differential equations and it has been successfully validated with using selected sociological data [7,15], yet all of them contained only symmetric relations. As was demonstrated in Ref. [16], our method is free from stable unbalanced (so-called 'jammed') states, which plague discrete methods based on the Monte-Carlo scheme [17,18]. The goal was not only to obtain the final partition, but also to simulate the actual dynamics of the cognitive process. It is important to note that the interpretation of the obtained partition depends on the context. In particular, the data analyzed in Ref. [15] contain just the list of attendance of a set of persons on their meetings, and the quality of relations is represented by the correlations of this attendance. In this case, the persons involved could be ignorant about any hostility or conflict in the group. In general, the method allows only to indicate two groups, with internal relations more tight (or friendly), than between the groups.

Our goal here is to generalize our former method to the case of non-symmetric relations. The motivation is twofold. The first is obvious: basically there is no reason to expect that the relations are symmetric, and actually – as is shown below – we are going to cope with data where they are not. The second aim is again to include the psychological process of direct reciprocity, one of the main mechanism of triggering cooperation between otherwise selfish agents [19]. With this generalized model, we are going to capture the dynamics of gender separation process, where both contacts in dyads and third-person reactions allow to reproduce a partition into two coherent groups, and where differences in patterns of behavior accepted in these groups stabilize the partition. The generalized equations are applied to the weighted data on mutual liking and aversion, collected in 37 classes in Mexican schools and four groups of teachers. The age of the respondents in the classes varied from 9 to 23. Each individual had to indicate five persons which she or he liked most, and grade them with 1–5 points; the same was asked about disliking, with negative grades. These data are presented in the form of non-symmetric matrices, one matrix per class. As a rule, these data do not provide a coherent partition of a class into two sets. This is done with the model differential equations. The outcome can be understood as the partition most close to the initial data. The analysis of the gender distribution of the obtained partition allows to indicate the classes where the gender is meaningful as a criterion of the partition. This is done by the calculation of the correlation between the proportion of males and females in the obtained groups.

The next section is devoted to the model equations. Part of this chapter is relegated to the Appendix. There we show that when the equations in their previous form are applied to non-symmetric data, they lead to jammed states, similarly to the non-deterministic algorithms [17,18]. Coming back to Section 2, we demonstrate, that the jammed states are removed in the generalized model, except the case when the rate of the new process is too small.

In Section 3 we provide the details on the collected data, which describe the relations between children in 37 school classes. Separate data sets for four groups of adults are taken as a reference point. In Section 4 we describe the numerical results of the application of the model equations to the relations between children. The analysis of the results allows to evaluate the level of the gender classification in each class. The last section is devoted to the discussion.

2. The model

In the initial formulation [8], the time evolution of an element $x(i, j)$ of the symmetric matrix of relations was given by

$$\frac{\mathrm{d}x(i,j)}{\mathrm{d}t} = G(x(i,j)) \sum_{k}^{N-2} x(i,k)x(k,j) \tag{1}$$

where the role of the term $G(x) = 1 - (x/R)^2$ was to limit the values of x to a prescribed range $(-R, R)$.

The idea behind Eq. (1) is as follows. Recall that the conditions of the removal of cognitive dissonance are as follows: (i) friend of my friend is my friend, (ii) friend of my enemy is my enemy, (iii) enemy of my friend is my enemy, (iv) enemy of my enemy is my friend [13]. Let us consider a particular relation x_{ij} between individuals i and j. For all their common neighbors k, the relations x_{ik} and x_{kj} are either of the same or different sign. In the former case, these two relations are either both