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Unfolding of the spectrum for chaotic and mixed systems

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HIGHLIGHTS

- Unfolding is necessary for analysis of spectral fluctuation.
- It is often done in terms of polynomial of degrees ranging from 3 to 20.
- The effect of polynomial degree on $P(S)$ and $\Sigma^2(L)$ which is particularly sensitive.
- We determine an optimum order for polynomial unfolding.

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ABSTRACT

Random Matrix Theory (RMT) is capable of making predictions for the spectral fluctuations of a physical system only after removing the influence of the level density by unfolding the spectra. When the level density is known, unfolding is done by using the integrated level density to transform the eigenvalues into dimensionless variables with unit mean spacing. When it is not known, as in most practical cases, one usually approximates the level staircase function by a polynomial. We here study the effect of unfolding procedure on the spectral fluctuation of two systems for which the level density is known asymptotically. The first is a time-reversal-invariant chaotic system, which is modeled in RMT by a Gaussian Orthogonal Ensemble (GOE). The second is the case of chaotic systems in which m quantum numbers remain almost undistorted in the early stage of the stochastic transition. The Hamiltonian of a system may be represented by a block diagonal matrix with m blocks of the same size, in which each block is a GOE. Unfolding is done once by using the asymptotic level densities for the eigenvalues of the m blocks and once by representing the integrated level density in terms of polynomials of different orders. We find that the spacing distribution of the eigenvalues shows a little sensitivity to the unfolding method. On the other hand, the variance of level number $\Sigma^2(L)$ is sensitive to the choice of the unfolding function. Unfolding that utilizes low order polynomials enhances $\Sigma^2(L)$ relative to the theoretical value, while the use of high order polynomial reduces it. The optimal value of the order of the unfolding polynomial depends on the dimension of the corresponding ensemble.

1. Introduction

Random matrix theory [1,2] provides a framework to describe the statistical properties of spectra for quantum systems, whose classical counterpart are chaotic. It models the Hamiltonian of the system by an ensemble of N-dimensional random matrices, conditioned by general symmetry constraints. For example, a time-reversal-invariant quantum system is represented by a GOE of random matrices. RMT is also used for the integrable system by representing the Hamiltonian as a real diagonal random matrix whose eigenvalues are drawn at random from a Gaussian, leading to Poisson Orthogonal Ensample (POE) fluctuations for these elements [3,4]. Nevertheless, it is well known that not all the regular systems have a Poissonian NNS distribution [5–7]. The two-dimensional harmonic oscillator is a classical example [8].

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In general RMT does not apply to the mixed systems for which the classical phase space has separate regions for regular and chaotic motion. Some models are introduced to apply RMT to mixed systems (see, e.g., [9,10] and references therein). These models allow studying the transition from integrability to chaos. One of these models assumes that some of the quantum numbers of the system are approximately conserved. In this case the spectrum is composed by a superposition of independent subspectra and the Hamiltonian is represented by a block-diagonal matrix [11].

RMT aims to explain the correlation between energy levels independently of the mean level spacing. For this purpose, it is more common to "unfold" the spectrum by means of transformation [12] involving the cumulated level density so that the mean level spacing is equal to one. The unfolding of the spectrum must be implemented to get rid of the non-universal properties (level density) and concentrate on the fluctuations properties of the spectrum, which display universal properties. In most of practical applications, the exact form of the cumulated density level is unknown. Unfolding is usually done by parameterizing the numerically obtained level density in the terms of a smooth function, typically a polynomial. The choice of the order of the unfolding polynomial introduces ambiguities at the unfolding procedure. Unfolding can also be done several other ways [13–15].

The aim of the present study is to quantify the randomness which may be present in spectral statistics due to the unfolding polynomial order. For this purpose, we discuss two special cases for which the spectral density is known. One of which is a simple GOE and the other has a composite spectrum of independent GOE sequences. We unfold the spectra of each model once by using the exact expression for the level density and once by approximating the density by polynomials of different degrees. We study the effect of different choice of the functional form of level density on spectral correlations. In particular, we quantify the short and long term correlations between levels by the nearest-neighbor spacing distribution $P(s)$ and the variance $\Sigma^2(L)$, respectively.

2. Unfolding of the spectrum

The main aim of RMT is to describe the fluctuations of the energy spectra. Before studying the fluctuations, we must separate the local level fluctuations from overall energy dependence of the level separation. The level density of a standard random matrix ensemble is not directly related to the physical level density of the investigated systems. Nevertheless, it is needed for the proper unfolding of the spectral fluctuation measures. Unfolding is usually done by calculating the cumulative spectral function $I(E)$ of the observed or computed spectra, which is defined as the number of levels below or at the energy E. This function is frequently referred to as the staircase function. It may be separated into an average part $I_{\rm ave}(E)$, whose derivative is the mean level density, and a fluctuating part $I_{\rm fluc}(E)$. $I_{\rm ave}(E)$ is calculated for each matrix of the ensemble by running spectral average. Whenever the functional form of mean level density $\rho(E)$ is known, the mean cumulative spectral density can be obtained,

$$I_{\rm ave}(E) = \int_{-\infty}^{E} {\rm d}E' \rho\left(E'\right). \tag{1}$$

The unfolded spectrum is formed by introducing a dimensionless energy variable

$$\varepsilon_i = I_{\rm ave}(E_i). \tag{2}$$

In this variable, the spectra possess mean level spacing unity everywhere. The spacing between two successive levels in the unfolded spectrum can be obtained by a Taylor expansion of $I_{\rm ave}(E)$, which yields

$$\begin{aligned}\varepsilon_{i+1} - \varepsilon_i &= (E_{i+1} - E_i)\,\frac{{\rm d}I_{\rm ave}\,(E_i)}{{\rm d}E_i} + \text{h.o.t.}\\ &= \frac{E_{i+1} - E_i}{D_i} + \text{h.o.t.}\end{aligned} \tag{3}$$

where $D_i = 1/\rho(E_i)$ is the mean level spacing in the vicinity of E_i. Eq. (3) suggests that the direct relation between the original and unfolded spectra is valid when the higher order terms can be neglected, which happens where the level density is slowly varying. This is usually true in the center of spectrum in most of the studied cases.

The mean level density can be estimated in some special cases. Spectra of billiards are unfolded in terms of level densities obtained from Weyl's semiclassical law [16], which relates the billiard area and circumference to the number of resonance frequencies below a given one. Nuclear spectra are often unfolded in terms of formulae for level density derived from the Fermi gas models [17].

2.1. Asymptotic level densities

If the system is modeled by GOE, the mean level density for infinitely large matrices is given by Wigner's semi-circle law [18]:

$$\rho_{\rm GOE}\,(N, E) = \begin{cases} \dfrac{2N}{\pi a^2}\sqrt{a^2 - E^2}, & \text{for } |E| \le a \\ 0, & \text{for } |E| > a \end{cases} \tag{4}$$