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Weber–Fechner relation and Lévy-like searching stemmed from ambiguous experiences

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HIGHLIGHTS

- We model the multi-agent based random walk algorithms.
- We show that Weber–Fechner relation in respect to step lengths can be emerged using our model.
- Power-law distributed move lengths are also achieved.
- The slope values differ by introducing ambiguous threshold-changings.

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ABSTRACT

Here, we show that an optimized Lévy-like walk ($\mu \approx 2.00$) and the Weber–Fechner law can be achieved in our new multi-agent based model that depends on step lengths. Weber–Fechner equation is strongly related to power-law. This equation is sometimes used in order to obtain power-law tailed distributions in observational levels. However, no study has reported how these two popular equations were achieved in micro or mechanistic levels. We propose a new random walk algorithm based on a re-valued algorithm, in which an agent has limited memory capacity, i.e., an agent has a memory of only four recent random numbers (limitation number). Using these random numbers, the agent alters the directional heuristic if the agent experiences moving directional biases. In this paper, the initial limitation number varies depending on the interaction among agents. Thus, agents change their limitation number and produce time delay in respect to rule change events. We show that slope values are variable compared with isolate foraging even though both indicate power-law tailed walks derived from Weber–Fechner equation.

1. Introduction

The fundamental problem with studying movement-strategy is how to achieve optimal searching without assuming a power-law distribution. A robust algorithm should not only optimize a random walk, but should also estimate how these flexible behaviors arise from the individual activities of each agent [1]. A Lévy distribution is defined as a process whereby an agent takes steps of length l at each time, where l is a probability density distribution with a power-law tail:

$$\mathrm{P}(l) \sim l^{-\mu} \tag{1}$$

with $1 < \mu \leqq 3$. A Lévy walk with $\mu \approx 2$ shows effective random searching in environments with randomly distributed food sites which are non-destructive [2,3]. For $\mu > 3$, the Lévy walk converges to Brownian motion.

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Does this flexible behavior emerge from intrinsic properties [4]? Recently, we proposed a random walk algorithm called re-valued algorithm, in which the agent has a limited memory capacity (i.e., only a memory of four recent random numbers) [5,6]. Using these random numbers, the agent changes its directional rule if the agent experiences moving directional biases. If the agent could observe its entire trajectory while maintaining unbiased random searching, then there would be no need to change the rule because some directional biases only occur probabilistically. However, in a re-valued algorithm, the agent is not able to observe its entire trajectory but is only aware of a limited trajectory. Therefore, to modify directional biases which are occupied by its current rule, the agent changes its directional rule by producing a correct or erroneous directional interpretation of a few random numbers, resulting in more biased movements that deviate from Brownian motion. Therefore, power-law tailed movements might be obtained from intrinsic properties in this model. However, even if an agent uses only its limited memory, non-deterministic uncertainness is needed beyond simple diffusion properties (see Ref. [6]). In this respect, the agent must hold also extrinsic properties in intrinsic ones.

Here, we introduce a new multi-agent random walk algorithm by using a re-valued algorithm as a base simulation model. In this paper, the limitation number of recent random numbers varies depending on the interactions between agents. Through local interactions, agents change their limitation number, which prompts them to delay the timing of rule-change. Thus, these interactions influence their rule changing indirectly. Here, we call these indirect interactions as "weak interactions". It is known that power-law tailed movements can be realized through interactions [7,8]. Our new model shows more optimal behaviors than base model's behaviors in respect to step lengths i.e. slope values (μ). We also show that Weber–Fechner relation is obtained from our model in respect to step lengths.

The Weber–Fechner relation is the linear relation between physical magnitudes of input stimulus and subjective perceived intensity and sometimes discussed as one of the origins of power-law distributions in phenomenological or observational levels [9–12]. However, no studies have reported how these two equations would be emerged from bottom-up or micro-level mechanisms. Here, we describe how these two equations are obtained on agent simulations by assuming that this approach could apply to observational results in which power-law is derived from Weber–Fechner equation. Subjective perceptions depending on physical magnitude of input information might be produced by agents' subjective interpretations. Therefore, deviation of Gaussian distributed errors of input information would emerge as subjective output. Later, we discuss the application possibility of this abstract model to actual cognitive phenomena to which Weber–Fechner equations and power-laws are strongly related. We also discuss biological significance related to our findings.

2. Materials and methods

2.1. Model description

We propose new random walk algorithm based on original re-valued (REV) algorithm [4,5]. In our new model, multi agents are in the same simulation field and through local interactions, every agent changes its own directional bias length. We describe the details of our new algorithm as follows.

Multi-re-valued (multi-REV) algorithm

In our multi-re-valued (multi-REV) random walk model, we set 500 agents in 500×500 grid size field. Each agent moves on two dimensional lattices with step length 1 on each time step. The kth agent has a memory at the tth step that allows the agent to access four random numbers, $R^k(t)$ to $R^k(t-3)$, and updates the interval in the multi-REV random walk, $I_s^k(t)$ for $s = 0, 1, 2, 3$, which is employed to a weighted dice. The $I_s^k(t)$ is changed using the four random numbers when the accumulation of the agent's directional moves exceeds a threshold number. The four intervals are initially set to

$$I_0^k(0) = [0.0, 0.25], \quad I_1^k(0) = [0.25, 0.5], \quad I_2^k(0) = [0.5, 0.75] \quad \text{and} \quad I_3^k(0) = [0.75, 1.0], \quad \text{for all } k\text{th agents.} \tag{2}$$

The next walk is defined by the following equations:

$$(x^k(t+1), y^k(t+1)) = (x^k(t)-1, y^k(t)), \quad \text{if } R^k(t) \in I_0^k(t); \quad = (x^k(t)+1, y^k(t)), \quad \text{if } R^k(t) \in I_1^k(t); \quad = (x^k(t), y^k(t)-1), \quad \text{if } R^k(t) \in I_2^k(t); \quad = (x^k(t), y^k(t)+1), \quad \text{if } R^k(t) \in I_3^k(t). \tag{3}$$

Thus, every trial starts as a simple random walk.

A directional move that consists of a series of the same move, such as $-x, -x, -x, \ldots$, is counted as follows:

$$\begin{aligned} &Experience^k(t+1) = Experience^k(t) + 1, \quad \text{if there exists } s \text{ in } \{0,1,2,3\} \text{ such that} \\ &\quad R^k(t-m) \in I_s^k(t) \text{ for all } m \text{ in } \{0, 1, \ldots, MAX^k\_u\}; \\ &Experience^k(t+1) = Experience^k(t), \quad \text{otherwise.} \end{aligned} \tag{4}$$

MAX^k_u is assigned to every agent as initial condition by choosing one value randomly such as

$$MAX^k\_u \in \{2, 3, 4, 5\}. \tag{5}$$