Physica A 438 (2015) 26–31

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

KdV–Burgers equation in the modified continuum model considering anticipation effect☆

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HIGHLIGHTS

- The driver's anticipation effect is introduced into the OV car-following model
- The KdV–Burgers equation and one solution are obtained, which is seldom investigated
- Numerical simulation is made to show the local cluster described by the model

ARTICLE INFO

Article history:
Received 20 September 2014
Received in revised form 21 April 2015
Available online 17 June 2015

Keywords:
Traffic flow
Continuum model
KdV–Burgers equation

ABSTRACT

The new continuum model mentioned in this paper is developed based on optimal velocity car-following model, which takes the drivers' anticipation effect into account. The critical condition for traffic flow is derived, and nonlinear analysis shows density waves occur in traffic flow because of the small disturbance. Near the neutral stability line, the KdV–Burgers equation is derived and one of the solutions is given. Numerical simulation is carried out to show the local cluster described by the model.

1. Introduction

Many models have been put forward to simulate different kinds of continuum traffic flow phenomena [1–8]. In 1955, Lighthill and Whitham [9,10] made the first contribution to continuum models and later Richards [11] drew the similar conclusion independently (for short, the LWR model). In this model, the relationship of the three basic parameters of the fluid is built by

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

where ρ, q, t, x, represent the density, flow, time and space, respectively. Only supplemented by the two equations of $q = \rho v$ and the equilibrium condition $v = v_e(\rho)$, Eq. (1) is a self-consistent model.

Through some follow-up work, the model above can be used to describe the majority of traffic flow phenomena. Liu et al. [12] considered that LWR model is an easy and convenient method but has a disadvantage. Some typical traffic

☆ Supported by the National Natural Science Foundation of China (Grant No. 11272160), the Scientific Research Fund of Zhejiang Provincial, China (Grant Nos. LY15A020007, LY15E080013, LY13A020007), the Natural Science Foundation of Ningbo (Grant Nos. 2014A610028, 2014A610022), and the K.C. Wong Magna Fund in Ningbo University, China.

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http://dx.doi.org/10.1016/j.physa.2015.05.014
0378-4371/

phenomena, which come down to the non-equilibrium traffic flow dynamics, such as, traffic hysteresis and phantom traffic jams, can hardly be explained.

In 1971, adding a dynamic equation to the continuity one, Payne [13] established a high-order continuum traffic flow model, which can describe the real traffic effectively. What is more, this model can show the nonlinear wave propagation characteristics and be used to analyze the traffic phenomena, such as the small disturbance instability and the stop-and-go traffic. The dynamic equation is:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\alpha}{\rho T}\frac{\partial \rho}{\partial x} + \frac{v_e - v}{T} \tag{2}$$

where T is relaxation time, and α is the anticipation coefficient.

In 1995, Daganzo [14] pointed out that, there are many advantages in Payne's model. Nevertheless, one of its characteristic speeds is greater than the macroscopic flow velocity, which shows a gas-like behavior. Based on the actual driving, it is widely acknowledged that vehicles respond only to frontal ones, which we call it anisotropic characteristics. According to the anisotropic characteristics of the high-order continuum models, scholars made more extensive and in-depth discussions. H.M. Zhang [15] put forward a new continuum traffic flow theory which overcomes vehicles driving backward problem happened in the high-order continuum models. Later H.M. Zhang [16] further discussed whether anisotropic property exists in multi-lane traffic.

In 1995, Bando et al. [17] proposed the optimal velocity model (for short, OVM), which can be used to simulate the qualitative characteristics of the actual traffic flow, such as the stop-and-go phenomenon, traffic instability and the congestion evolution and so on. The model is:

$$\dot{v}_n(t) = a\left[V(\Delta x_n(t)) - v_n(t)\right] \tag{3}$$

in which the item $V(\Delta x_n(t))$ means an optimal velocity depended on space headway $\Delta x_n(t)$. But the traffic phenomena described by Treiber et al. [18] cannot be explained by OVM. Namely, if the preceding cars are much faster than the following ones, the vehicle would not brake, even if its headway is smaller than the safe distance. In order to solve this problem, Jiang et al. [5] proposed a full velocity difference model (for short, FVDM). The formula of the FVDM is

$$\frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t} = \kappa\left[V(\Delta x) - v_{n+1}(t)\right] + \lambda\Delta v. \tag{4}$$

Based on FVDM, Jiang et al. [19] derived an anisotropic macroscopic continuum model—the velocity gradient model, which allows the vehicles' speed deviate from speed–density relationship and can be used to analyze the stop-and-go and the small disturbance instability traffic phenomena. The model is:

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t} = \frac{V_e(\rho(x,t)) - v(x,t)}{T} + \frac{\Delta}{\tau}\frac{\partial v}{\partial x} \tag{5}$$

where $c_0 = \Delta/\tau$ means disturbance propagation speed.

Recently, the anticipation effect is considered in lattice hydrodynamic models by Peng et al. [20–22]. We will investigate the problem from continuum model.

The rest of this paper is organized as follows. In Section 2, the new model is derived and then the stability analysis is used. Through nonlinear analysis, the KdV–Burgers equation is derived in Section 3 and the simulation is carried out in Section 4. Finally, the conclusion is given in Section 5.

2. Model and stability analysis

As we know, the time lag produced by the driver's reflection and the transmission of automotive crankshaft system exists in the real traffic. But this influence is ignored in most previous investigations. We consider the drivers' expected effect and get:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = a\left[V(\Delta x_n(t) + T\Delta v_n) - v_n(t)\right] \tag{6}$$

in which T means the anticipation time and the term $T\Delta v_n$ represents distance influenced by the expected time.

Through Taylor expansion, we obtain

$$V(\Delta x_n(t) + T\Delta v_n) = V(\Delta x_n(t)) + T\Delta v_n V'(\Delta x_n(t)) \tag{7}$$

where $\Delta v_n = v(x+\Delta, t) - v(x,t)$. Then we use the following relation to rewrite the above micro variables into macro ones:

$$\begin{aligned} &v_n(t) \to v(x,t), \qquad v_{n+1}(t) \to v(x+\Delta, t) \\ &V(\Delta x_n(t)) \to V_e(\rho), \qquad V'(\Delta x_n(t)) \to \bar{V}'(h) \end{aligned} \tag{8}$$

where Δ represents the distance between two adjacent vehicles. Through the density ρ and the mean headway $h = 1/\rho$, we define the equilibrium speed $V_e(\rho)$ and have $\bar{V}'(h) = -\rho^2 V_e'(\rho)$. Considering the continuous conservation equation,